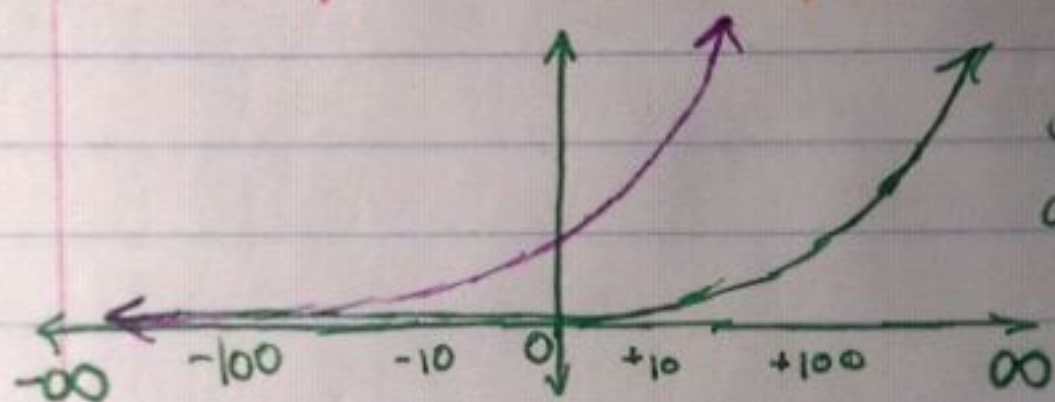


# Exponential function Review

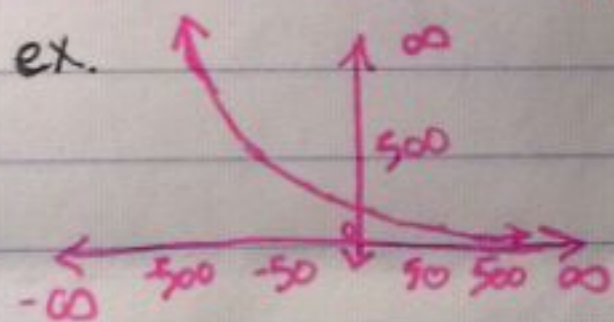
$$f(x) = a(b)^x$$

y-int  $\downarrow$  base multiplier,  $b > 0, b \neq 1$



as  $x \rightarrow -\infty, y \rightarrow \emptyset$  ex  $y = 2^x$   
as  $x \rightarrow \infty, y \rightarrow \infty$   $y = 10^x$   
 $y = e^x$

When the base multiplier  $b$  is a number between  $0$  and  $1$  ( $0 < b < 1$ ), it will be a decreasing exponential



as  $x \rightarrow -\infty, y \rightarrow \infty$   
as  $x \rightarrow \infty, y \rightarrow \emptyset$

## Change of bases

$$\log_{\Delta} M = \frac{\log M}{\log \Delta}$$

where  $\Delta \neq 10, \Delta \neq e$

$$2^x = 5 \rightarrow x = \frac{\log 5}{\log 2}$$
$$\log 2^x = \log 5$$
$$x \log 2 = \log 5 \quad x \approx 2.322$$

## Product Rule of Logs

$$\log_b(xyz) = \log_b x + \log_b y + \log_b z$$

## Quotient Rule

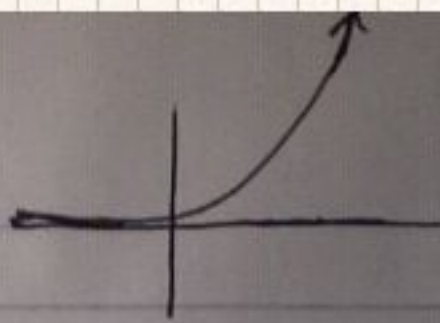
$$\log_b \frac{x}{z} = \log_b x - \log_b z$$

## Power Rule of Log

$$\log_b x^z = z \log_b x$$



$$f(x) = \frac{1+x}{e^{\cos x}}$$



One to One functions pass both the vertical line and horizontal line tests.

Given that a function  $f(x)$  that is one-to-one, it has an inverse function,  $f^{-1}(x)$

- 1)  $f \circ f^{-1}(x) = x$
- 2) To find a function's inverse, swap  $x$  for  $y$  and solve for  $y$ .
- 3) The graph of a function and its inverse are ~~sym~~ symmetrical with respect to origin.

Given  $f(x) = 3x - 5$ , find  $f^{-1}(x)$

- 1) Rewrite  $f(x)$  as  $y =$
- 2) solve for  $x$
- 3) swap  $x$  +  $y$
- 4) Rewrite  $y =$  as  $f^{-1}(x)$

$$2 \log(2x!) + \log x - 4 \log$$

• Vertical, Horizontal, diagonal Asymptote

if  $F(x) = b^x$ , where  $b > 0, b \neq 1$ , its inverse is

$$F^{-1}(x) = \log_b x$$

if  $\log_b 1 = 0$  if  $\log_b b = 1$  if  $b^{\log_b x} = x$

$$F(x) = \log_{10}(2x+3)$$

$$y = \log_{10}(2x+3)$$

$$10^y = 2x+3$$

$$10^y - 3 = 2x$$

$$\log_5(x-5) + \log_5(x+4) + \log_5(x+4)^2$$

$$\frac{10^y - 3}{2} = x \quad \frac{10^x - 3}{2} = F^{-1}(x)$$

$$\log_5(x-5) + 2 \log_5(x+4)$$

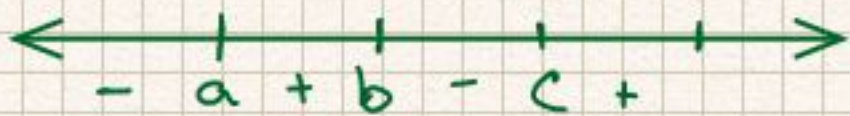


$$27) f(x) = \frac{x^2}{1+2x}$$

$$f'(x) = \frac{2x^2 + 2x}{(1+2x)^2}$$

$$\begin{aligned} f''(x) &= \frac{(4x+2)(1+2x)^2 - (2x^2+2x)(4+8x)}{(1+2x)^4} \\ &= \frac{2(2x+1)(1+2x)^2 - (2x^2+x)4(1+2x)}{(1+2x)^4} \\ &= \frac{2(2x+1)}{(1+2x)^3} = \frac{2}{(2x+1)^3} \end{aligned}$$

$$f'(x)$$



Given  $f(x)$  is differentiable, on the interval  $f'(x) > 0$ ,  $f(x)$  is increasing. If  $f'(x) < 0$ ,  $f(x)$  is decreasing. Where  $f'(x) = 0$ ,  $f(x)$  will have a max or a minimum. On the interval  $f''(x) > 0$ ,  $f(x)$  is concave up. If  $f''(x) < 0$ ,  $f(x)$  is concave down.

$$1) \lim_{z \rightarrow 0} \frac{\sin(10z)}{z} = \frac{10}{1 \cdot 10}$$

$$\lim_{\Delta \rightarrow 0} \frac{\sin \Delta}{\Delta} = 1$$

$$2) \lim_{\alpha \rightarrow 0} \frac{\sin(12\alpha)}{\sin(5\alpha)} = \frac{12}{5}$$

given  $f(x) = \sin x$ ,  
find  $f'(x)$

$$\begin{aligned} f(x+h) &= \sin(x+h) \quad f(x) = \sin x \\ &= \sin x \cos h + \cos x \sin h - \sin x \end{aligned}$$

$$3) \lim_{x \rightarrow 0} \frac{\cos(4x) - 1}{x} \cdot 4 = 0 \cdot 4 = 0$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$



Given  $f(x) = \tan x$ .

find  $f'(x)$

$$\tan x = \frac{\sin x}{\cos x}$$

let  $g(x) = \sin x, g'(x) = \cos x$

let  $h(x) = \cos x, h'(x) = -\sin x$

$$= \frac{[\cos^2 x + \sin^2 x]}{\cos^2 x} = \frac{1}{\cos^2 x} \Rightarrow \boxed{\sec^2 x} = f'(\tan x)$$

$$\frac{\cos x (\cos x) - [\sin x (-\sin x)]}{(\cos x)^2}$$

$f(x) = \cot(x)$

$$\cot(x) = \frac{\cos(x)}{\sin(x)}$$

$f(x) = \cos(x) \rightarrow f'(x) = -\sin(x)$

$f(x) = \tan(x) \rightarrow f'(x) = \sec^2(x)$

$g(x) = \sin(x) \rightarrow g'(x) = \cos(x)$

$f(x) = \cot(x) \rightarrow f'(x) = -\csc^2(x)$

$$\frac{\sin(x)(-\sin(x)) - [\cos(x)(\cos(x))]}{\sin^2(x)}$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} \Rightarrow \frac{-1(\sin^2 x + \cos^2 x)}{\sin^2(x)} \Rightarrow \frac{-1}{\sin^2(x)} \Rightarrow \boxed{-\csc^2(x)}$$

$$f(x) = \sec x = \frac{1}{\cos x} \rightarrow \frac{[\cos x(0) - (1)(-\sin x)]}{\cos^2(x)} = \frac{\tan(x)}{\cos(x)} = \boxed{\tan(x) \sec(x)}$$

let  $f(x) = 1, f'(x) = 0$

let  $g(x) = \cos x, g'(x) = -\sin x$

$$f(x) = \csc(x) = \frac{1}{\sin x} = \frac{-\cos(x)}{\sin^2(x)} = \frac{-\cos(x)}{\sin(x)} \cdot \frac{1}{\sin(x)} = \boxed{-\cot(x) \csc(x)}$$

$f(x) = 1, f'(x) = 0$

$g(x) = \sin(x), g'(x) = \cos(x)$

Given  $f(x)$  &  $g(x)$ , find  $f[g(x)]$

①  $f(x) = \sqrt{x}, g(x) = 2x-1$

$$f \circ g(x) = \sqrt{2x-1}$$

②  $f(x) = \sin(x), g(x) = \sqrt{x}$

$$f \circ g(x) = \sin \sqrt{x}$$

③  $f(x) = e^x, g(x) = \tan(x)$

$$f \circ g(x) = e^{\tan(x)}$$



Given

$$F = (f \circ g)(x) = f[g(x)]$$

$$\text{then } \frac{dF}{dx} = [F'(x)g(x)] \cdot g'(x)$$

$$F(x) = \sqrt{\tan x} = \tan x^{1/2}$$

$$\text{let } f(x) = x^{1/2}, g(x) = \tan x$$

$$f'(x) = \frac{1}{2\sqrt{x}} \quad g'(x) = \sec^2 x$$

$$f'(g(x)) \cdot g'(x) \text{ or } \frac{1}{2} \tan x^{-1/2} \cdot \sec^2 x$$

$$\frac{1}{2\sqrt{\tan x}} \cdot \sec^2 x$$

$$F(x) = (2x^2 - 5x + 10)$$

$$\text{let } f(x) = x^{99}, f'(x) = 99x^{98}$$

$$\text{let } g(x) = 2x^2 - 5x + 10, g'(x) = 4x - 5$$

$$\rightarrow 99(2x^2 - 5x + 10)^{98} \cdot (4x - 5)$$

$$F(x) = \sqrt{x^2 + 1}$$

$$\text{let } f(x) = \sqrt{x}, f'(x) = \frac{1}{2\sqrt{x}}$$

$$\text{let } g(x) = x^2 + 1, g'(x) = 2x$$

$$f'(g(x)) \cdot g'(x) \rightarrow \frac{1}{2\sqrt{x^2+1}} \cdot 2x = \frac{x}{\sqrt{x^2+1}} = \frac{x\sqrt{x^2+1}}{x^2+1}$$

$$F(x) = \sin(x^2)$$

$$f(x) = \sin(x) \rightarrow f'(x) = \cos(x)$$

$$g(x) = x^2 \rightarrow g'(x) = 2x$$

$$\cos(x^2) \cdot 2x = 2x \cos(x^2)$$

$$F(x) = \sin^2(x) = [\sin x]^2$$

$$f(x) = x^2 \quad f'(x) = 2x$$

$$g(x) = \sin(x) \quad g'(x) = \cos(x)$$

$$2(\sin x) \cdot \cos x$$

$$\textcircled{8} (4x - x^2)^{100}$$

$$f(x) = x^{100} \quad f'(x) = 99$$

$$g(x) = 4x - x^2 \quad g'(x) = -2x + 4$$

$$100(4x - x^2)^{99} \cdot (-2x + 4)$$

$$= 200(-x + 2)(4x - x^2)^{99}$$

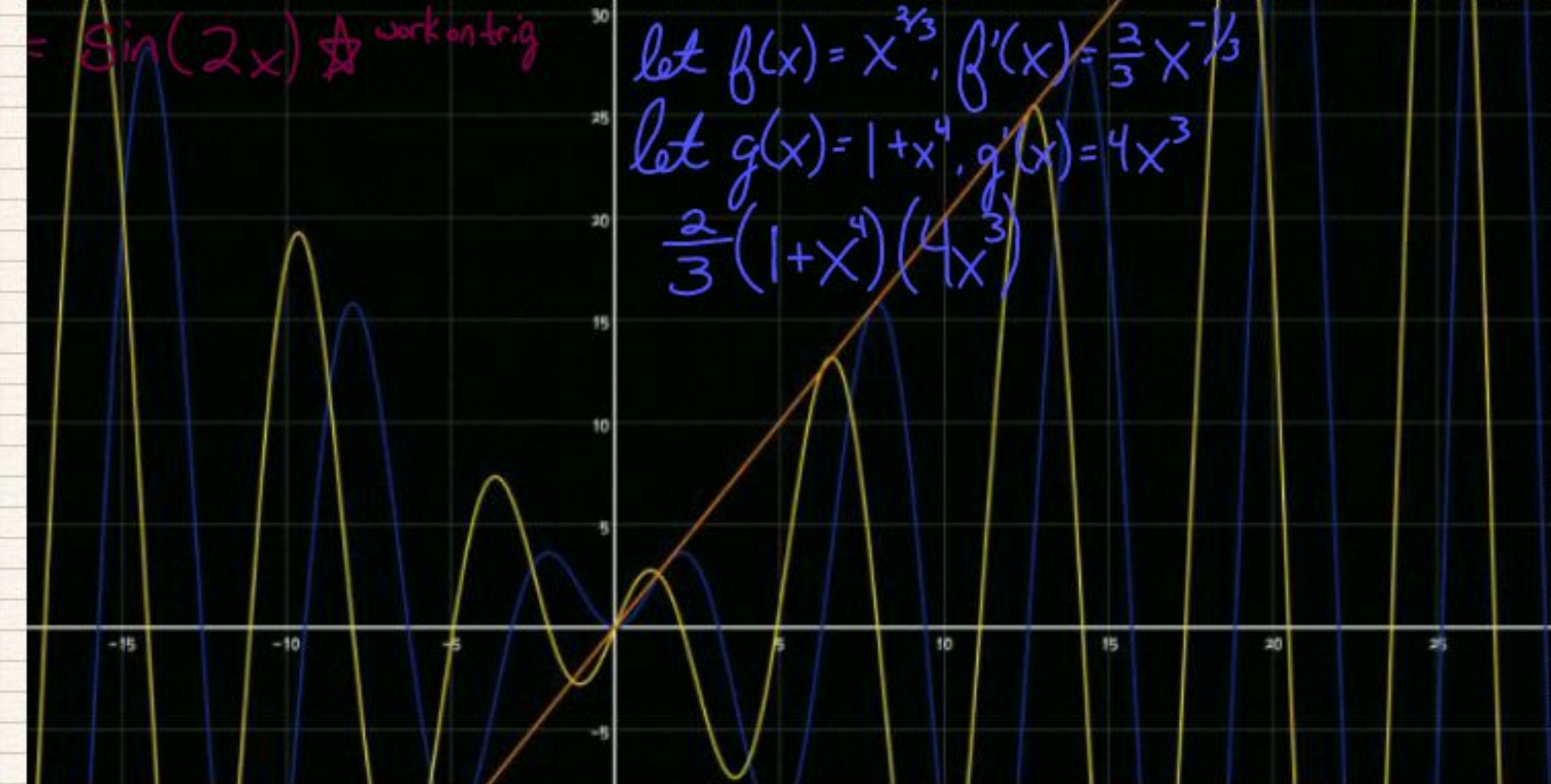
$$\textcircled{10} F(x) = (1 + x^4)^{2/3}$$

$$\text{let } f(x) = x^{2/3}, f'(x) = \frac{2}{3} x^{-1/3}$$

$$\text{let } g(x) = 1 + x^4, g'(x) = 4x^3$$

$$\frac{2}{3} (1 + x^4)^{-1/3} (4x^3)$$

$$= \sin(2x) \star \text{work on trig}$$





$f(x) = \sin(x^2 - 4)$      $\frac{d}{dx} \left[ \sqrt[3]{\cos(e^{\sec x})} \right]$   
 let  $f(x) = \sin x$      $= \frac{d}{dx} \cos(e^{\sec x})^{1/3}$   
 let  $g(x) = x^2 - 4$      $= \frac{-1}{3} [\cos(e^{\sec x})]^{-2/3} \sin(e^{\sec x}) (e^{\sec x}) (\sec x + \tan x)$   
 $\frac{d}{dx} f(g(x)) = f'(g(x)) (g'(x))$

$y = \sqrt{\sin(e^{\cos x})}$   
 $= [\sin(e^{\cos x})]^{1/2}$

Understand the problem  
 Come up with a plan  
 Execute the plan  
 Check your work

$= \frac{1}{2\sqrt{\sin(e^{\cos x})}} \cdot (\cos(e^{\cos x}) \cdot e^x (-\sin x))$

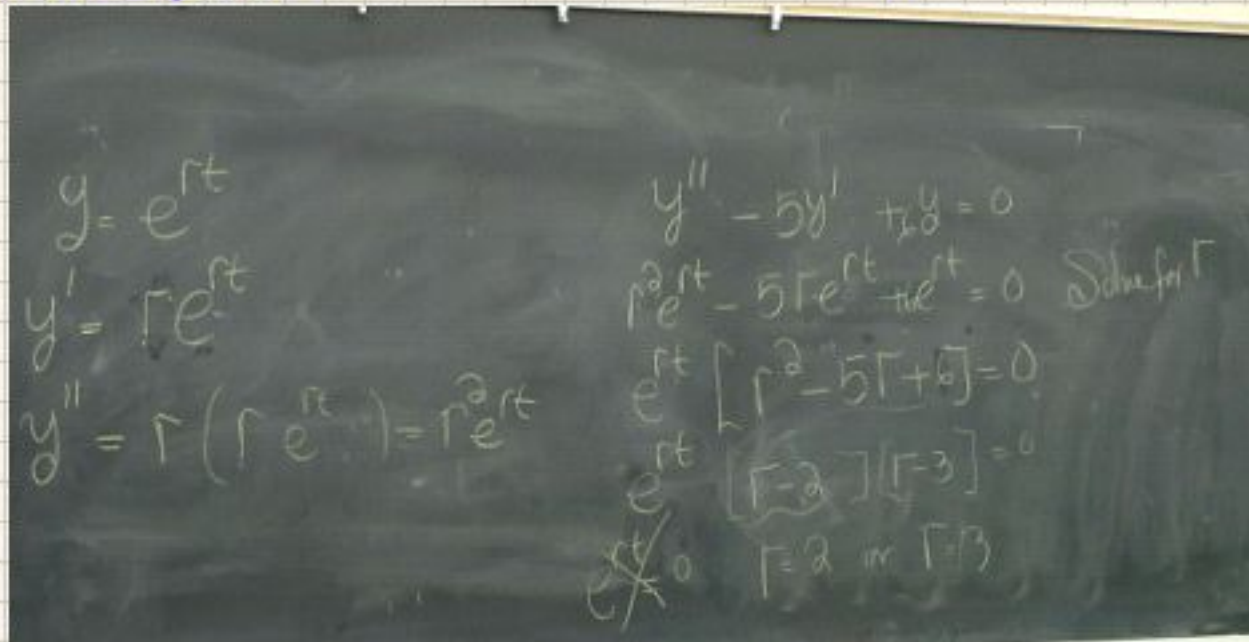
For what values of  $r$  does  $y = e^{rt}$  satisfy the differential equation  
 $y'' - 5y' + 6y = 0$   
 $y = e^{rt}$      $y' = r e^{rt}$      $y'' = r(r e^{rt})$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dx}{dt}$

$x = \frac{1}{2}t$      $y = t^2 - 3$   
 $x' = \frac{1}{2}$      $y' = 2t$   
 $\frac{2t}{\frac{1}{2}} = 2t(2) = 4t$

$y^2 + x^2 = 1$   
 $y^2 = 1 - x^2$   
 $y = \pm \sqrt{1 - x^2}$

Implicit  
 $5y^2 + \sin y = x^2$

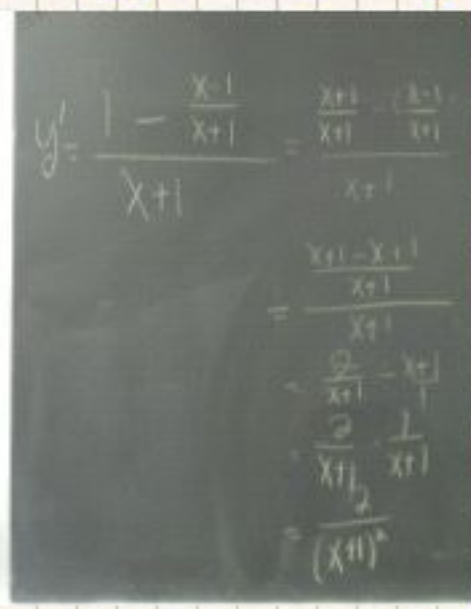
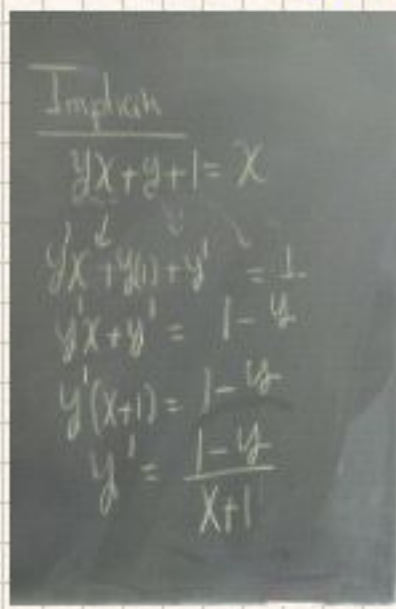


Implicit

Explicit

$yx + y + 1 = x$   
 $y'x + y(1) + y' = 1$   
 $y'x + y' = 1 - y$

$yx + y + 1 = x$   
 $yx + y = x - 1$   
 $y(x + 1) = x - 1$   
 $y = \frac{x - 1}{x + 1}$





$$\frac{d}{dx} [x^n] = nx^{n-1}$$

$$\frac{d}{dx} [u^n] = nu^{n-1} \frac{du}{dx}$$

$u = f(x)$

$$\frac{d}{dx} [f(x)g(x)] = f'(x)g(x) + g'(x)f(x)$$

$$\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{g(x)[f'(x)] - [f(x)g'(x)]}{[g(x)]^2}$$

$$\frac{d}{dx} [f^{-1}(x)] = \frac{1}{f'[f^{-1}(x)]}$$

let  $y = f^{-1}(x)$

$$f(y) = f[f^{-1}(x)]$$

$$f(y) = x$$

$$f'(y) = 1$$

$$f(x) = \ln x \rightarrow f'(x) = \frac{1}{x}$$

$$f(x) = \log_b x \rightarrow f'(x) = \frac{1}{x \ln b}$$

#2  $f(x) = x \ln x - x$

$$\left[ x \left( \frac{1}{x} \right) + \ln x \right] - 1$$

$$\frac{x}{x} + \ln x - 1 = \ln x$$

$$f(x) = \ln[\sin^2 x]$$

$$= \ln[(\sin x)^2]$$

$$\frac{d}{dx} \rightarrow \frac{1}{\sin^2 x} \cdot 2 \sin x \cdot \cos x$$

$$\frac{2 \sin x \cos x}{[\sin x]^2} \rightarrow$$

$$F(y) = y \ln[1+e^y]$$

$$F'(y) = 1 \cdot \ln[1+e^y] + \frac{y \cdot e^y}{1+e^y}$$

$$a^y = x \Leftrightarrow \log_a x = y$$

$$1. \log_b x = y \iff b^y = x$$

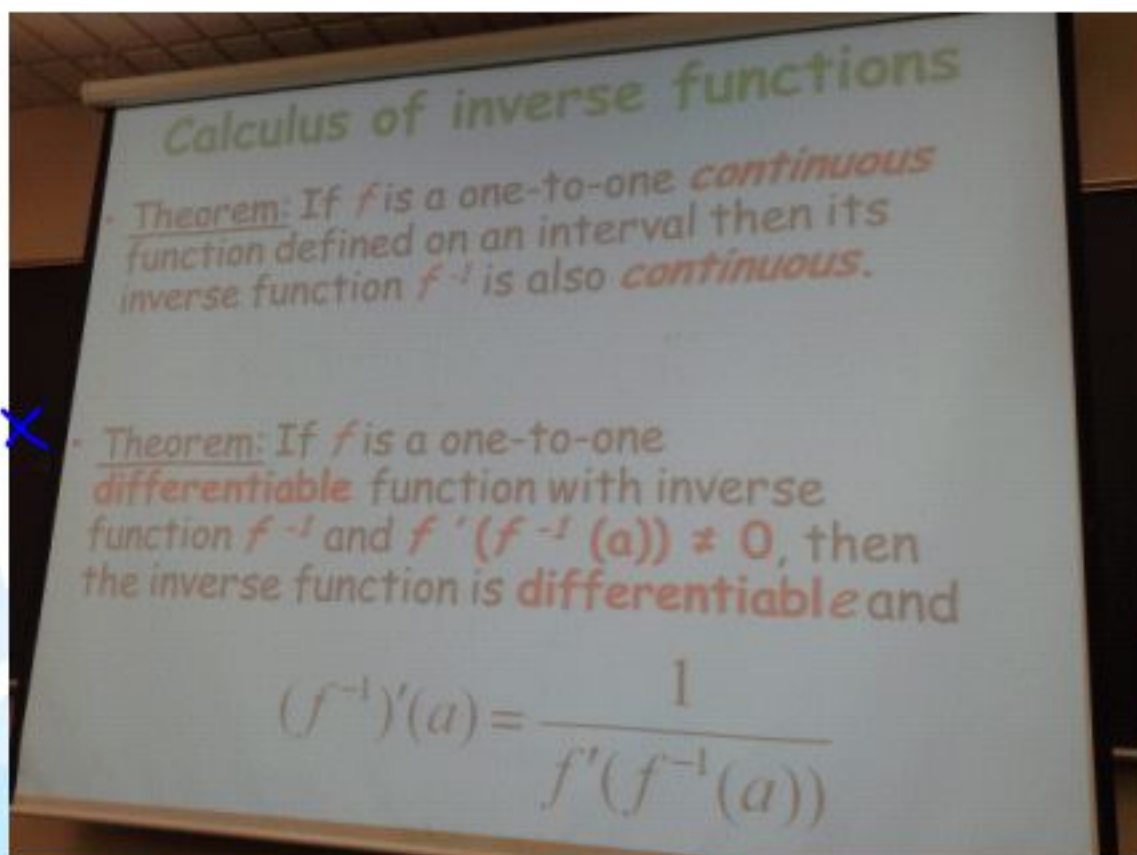
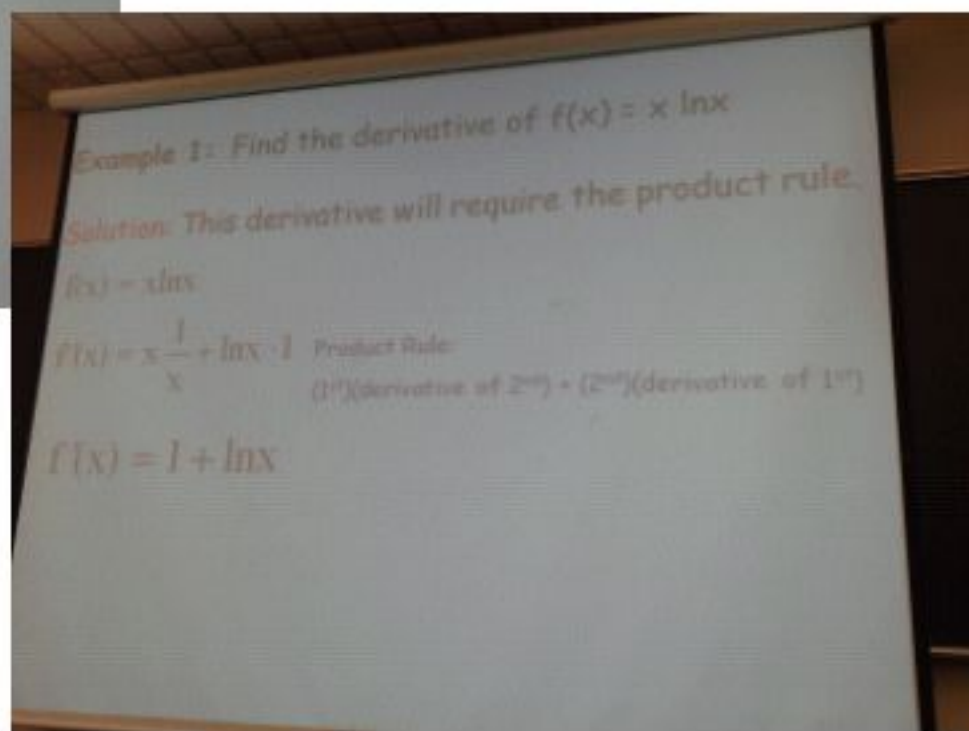
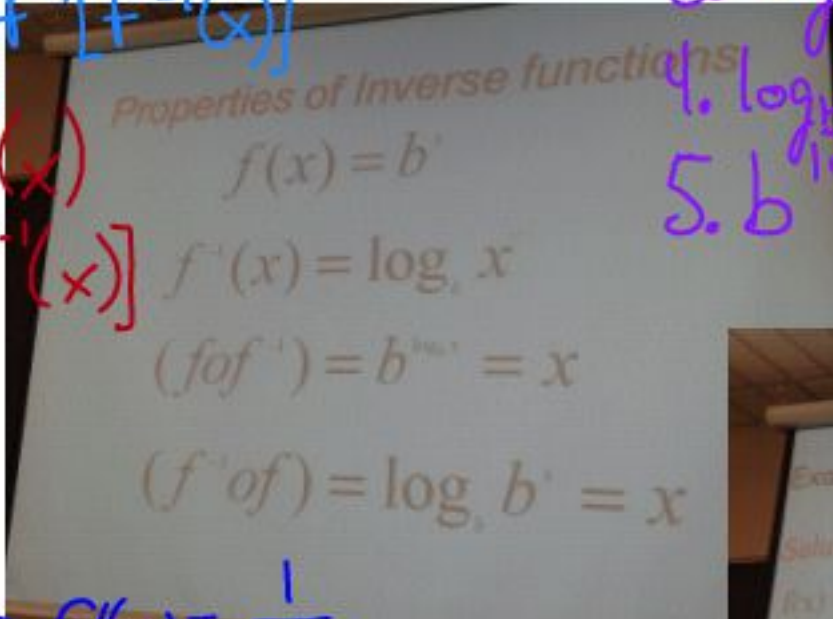
$$2. \log_b 1 = 0$$

$$3. \log_b M^p = p \log_b M$$

$$3. \log_b b = 1$$

$$4. \log_b b^x = x$$

$$5. b^{\log_b x} = x$$





$$g(x) = \frac{\ln x}{x}$$

$$y = \ln[(x^2+1)(x^3+2)^6]$$

$$g(t) = \ln[t^2 \cdot e^{-t^2}]$$

$$= \ln[t^2] + \ln[e^{-t^2}]$$

$$= 2\ln[t] - t^2 \ln e$$

$$g(t) = 2\ln t - t^2$$

$$g'(t) = 2 \cdot \frac{1}{t} - 2t$$

$$g''(t) = \frac{2}{t^2} - 2t$$

Example 4: Differentiate  $y = \ln[(x^2+1)(x^3+2)^6]$

Solution: There are two ways to do this problem. One is easy and the other is more difficult.

The difficult way:

$$y' = \frac{\frac{d}{dx}(x^2+1)(x^3+2)^6}{(x^2+1)(x^3+2)^6}$$

$$= \frac{(2x)(x^3+2)^6 + (x^2+1)6(x^3+2)^5(3x^2)}{(x^2+1)(x^3+2)^6}$$

$$= \frac{2x(x^3+2)^6 + 6(x^2+1)(3x^2)(x^3+2)^5}{(x^2+1)(x^3+2)^6}$$

$$= \frac{2x(x^3+2)^6 + 18x^2(x^3+2)^5}{(x^2+1)(x^3+2)^6}$$

Differentiate  $y = \ln[(x^2+1)(x^3+2)^6]$

The easy way requires that we simplify the log using some of the expansion properties.

$$y = \ln[(x^2+1)(x^3+2)^6] = \ln(x^2+1) + \ln(x^3+2)^6 = \ln(x^2+1) + 6\ln(x^3+2)$$

Now using the simplified version of  $y$  we find  $y'$ :

$$y = \ln(x^2+1) + 6\ln(x^3+2)$$

$$y' = \frac{2x}{x^2+1} + \frac{6(3x^2)}{x^3+2}$$

Now get a common denominator:

$$y' = \frac{2x(x^3+2)}{(x^2+1)(x^3+2)} + \frac{6(3x^2)(x^2+1)}{(x^3+2)(x^2+1)}$$

### Logarithmic Differentiation

The calculation of derivatives of complicated functions involving products, quotients, or powers can often be simplified by taking logarithms.

- **Step 1:** Take natural logarithms of both sides of an equation  $y = f(x)$  and use the properties of logarithms to simplify.
- **Step 2:** Differentiate implicitly with respect to  $x$
- **Step 3:** Solve the resulting equation for  $y'$

$$f(x) = (2x-1)^2$$

$$= 4x^2 - 4x + 1$$

$$\frac{df}{dx} = 8x - 4$$

$$f(x) = (2x-1)^2$$

$$y = (2x-1)^2$$

$$\ln y = \ln(2x-1)^2$$

$$\ln y = 2 \ln(2x-1)$$

$$\frac{1}{y} y' = 2 \cdot \frac{1}{2x-1} \cdot 2$$



Example 7: Differentiate  $y = x(x+1)(x^2+1)$

Solution: Although this problem could be easily done by multiplying the expression out, I would like to introduce to you a technique which you can use when the expression is a lot more complicated.

Step 1: Take the ln of both sides.

$$\ln y = \ln x(x+1)(x^2+1)$$

Step 2: Expand the complicated side.

$$\ln y = \ln x(x+1)(x^2+1)$$

$$\ln y = \ln x + \ln(x+1) + \ln(x^2+1)$$

Step 3: Differentiate both side (implicitly for ln y)

$$\ln y = \ln x + \ln(x+1) + \ln(x^2+1)$$

$$\frac{y'}{y} = \frac{1}{x} + \frac{1}{x+1} + \frac{2x}{x^2+1}$$

Continue to simplify.

$$y' = \left[ \frac{x(x+1)(x^2+1)}{x} + \frac{x(x+1)(x^2+1)}{x+1} + \frac{2x(x(x+1)(x^2+1))}{x^2+1} \right]$$

$$y' = (x+1)(x^2+1) + x(x^2+1) + 2x(x+1)$$

$$y' = (x^2+x^2+x+1) + (x^3+x+1) + 2x^2+2x$$

$$y' = 4x^2 + 3x + 2$$

Consider the function  $y = x^x$ .  
What is that minimum point?



Not a power function nor an exponential function.

This is the graph: domain  $x > 0$

What is that minimum point?

Recall to find a minimum, we need to find the first derivative, find the critical numbers and use either the First Derivative Test or the Second Derivative Test to determine the extrema.

To find the derivative of  $y = x^x$ , we will take the ln of both sides first and then expand.

$$y = x^x$$

$$\ln y = \ln x^x$$

$$\ln y = x \ln x$$

$$\ln y = x \ln x$$

$$\frac{y'}{y} = x \cdot \frac{1}{x} + \ln x \cdot 1$$

$$\frac{y'}{y} = 1 + \ln x$$

$$y' = y(1 + \ln x)$$

$$y' = x^x(1 + \ln x)$$

To find the critical numbers, set  $y' = 0$  and solve for x.

$$y' = y(1 + \ln x) = x^x(1 + \ln x)$$

$$0 = x^x(1 + \ln x)$$

$$0 = 1 + \ln x$$

$$-1 = \ln x$$

$$e^{-1} = x$$

$$e^{-1} = \frac{1}{e} \approx .367...$$



Find the derivative:  $y = x^{\cos x}$

Find the derivative  
we will take the ln of  
both sides first  
and then expand

$$y = x^{\cos x}$$

$$\ln y = \cos x \cdot \ln x$$

$$\frac{y'}{y} = \cos(x) \frac{1}{x} + \ln x \cdot (-\sin(x))$$

Now, to find the  
derivative  
we differentiate  
both sides  
implicitly

$$\frac{y'}{y} = \frac{\cos(x) - x \ln(x) \cdot \sin(x)}{x}$$

$$y' = y \left( \frac{\cos(x) - x \ln(x) \cdot \sin(x)}{x} \right)$$

$$y' = x^{\cos(x)} \left( \frac{\cos(x) - x \ln(x) \cdot \sin(x)}{x} \right)$$

Ch 3.7

Text: 25, 29, 33, 37, 43, 46

Worksheet

# Pick any 6 of your choice

3

$$h(t) = \frac{\sqrt{5t+8} \sqrt[3]{1-9\cos(4t)}}{\sqrt[4]{t^2+10t}}$$

$$\ln h(t) =$$

$$y = \sqrt{x}^x$$

- $\ln y = \ln[\sqrt{x}]^x$
- $\ln y = x \ln[\sqrt{x}]$

$$\frac{y'}{y} = 1 \cdot \ln[\sqrt{x}] + x \cdot \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$$

$$\frac{y'}{y} = \ln\sqrt{x} + \frac{x}{2x}$$

$$y' = y \left[ \ln\sqrt{x} + \frac{1}{2} \right]$$

$$y' = \sqrt{x}^x \cdot \left[ \frac{2\ln\sqrt{x} + 1}{2} \right]$$

$$h = \frac{\sqrt{5t+8} \sqrt[3]{1-9\cos(4t)}}{\sqrt[4]{t^2+10t}}$$

$$\ln h = \ln\sqrt{5t+8} + \ln\sqrt[3]{1-9\cos(4t)} - \ln\sqrt[4]{t^2+10t}$$

$$\ln h = \frac{1}{2}\ln(5t+8) + \frac{1}{3}\ln(1-9\cos(4t)) - \frac{1}{4}\ln(t^2+10t)$$

$$\frac{h'}{h} = \frac{5}{2(5t+8)} + \frac{9 \cdot 2 \sin(4t) \cdot 4}{3(1-9\cos(4t))} - \frac{(2t+10)}{4(t^2+10t)}$$

$$\frac{h'}{h} = \left[ \frac{5}{2(5t+8)} + \frac{36 \sin(4t)}{3(1-9\cos(4t))} - \frac{2(t+5)}{4(t^2+10t)} \right]$$

$$h' = \frac{\sqrt{x}}{y} \left[ \frac{5}{2t+16} + \frac{12 \sin(4t)}{1-9\cos(4t)} - \frac{t+5}{2t^2+20t} \right]$$



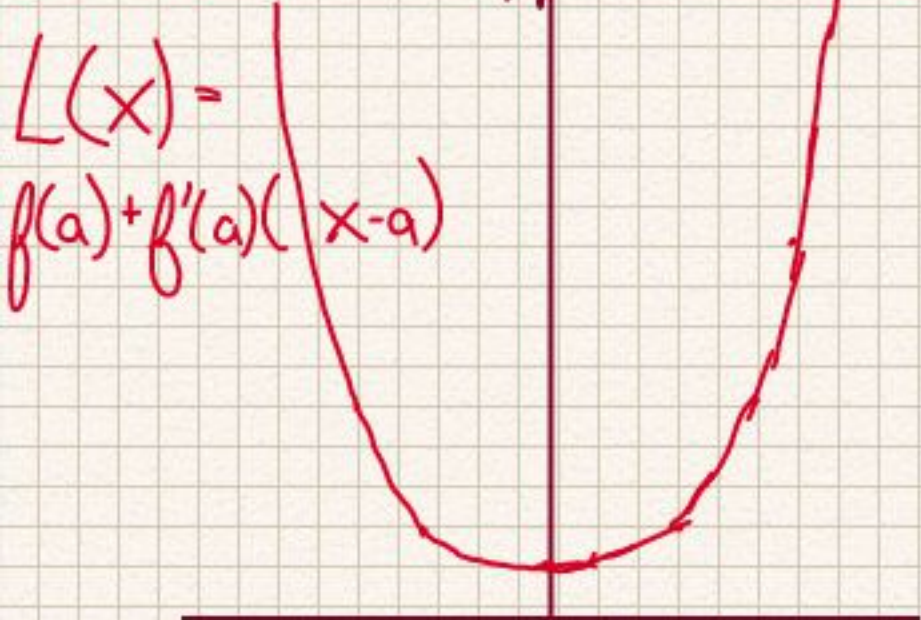
$$Q(v) = \frac{2}{(6 + 2v - v^2)^4}$$

$$= 2(6 + 2v - v^2)^{-4}$$

So far, we know  
3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7

Now we skip 3.8 & begin

### 3.9 Linear Approximation



⑪ Find  $y'' = \frac{d^2 y}{dx^2} = ?$   
given by  $xy' = 1$

• Use the derivative  
Previously we have used derivatives to find

- the equations of the tangent and normal lines to a function at a given point
- the velocity function given the displacement function
- the acceleration function given the velocity function

• Now we will learn, how to use derivatives

- To find the linearization (a.k.a. linear approximation) of a function at a point

Preview of things to come ...

• Example:  $f(x) = \sin x$  @  $a = \pi/4$

$f(\pi/4) = \sin(\pi/4) = \frac{\sqrt{2}}{2}$   
 $f'(\pi/4) = \cos(\pi/4) = \frac{\sqrt{2}}{2}$   
 $f''(\pi/4) = -\sin(\pi/4) = -\frac{\sqrt{2}}{2}$   
 $f'''(\pi/4) = -\cos(\pi/4) = -\frac{\sqrt{2}}{2}$

$\sin x \approx \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}(x - \frac{\pi}{4}) - \frac{\sqrt{2}}{4}(x - \frac{\pi}{4})^2 - \frac{\sqrt{2}}{12}(x - \frac{\pi}{4})^3$

Given  $f(x) = \sin(x)$ , find the linear approximation at  $\pi/4$

$$L(x) = f\left(\frac{\pi}{4}\right) + f'\left(\frac{\pi}{4}\right)\left(x - \frac{\pi}{4}\right)$$

$$L(x) = f\left(\frac{\pi}{4}\right) + f'\left(\frac{\pi}{4}\right)\left(x - \frac{\pi}{4}\right)$$



---

$f(x) = \sin x \rightarrow f\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$   
 $f'(x) = \cos x \rightarrow f'\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$$L(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$$

$\sin(x) \approx \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$




 $x = 10 \text{ cm}$   
 $\frac{dx}{dx} = \pm \frac{1}{5} \text{ cm}$

3.9 syllabus.  
 Takehome Quiz

Find the estimated error in area

$A = x^2$     $A(x) = x^2$     $\frac{dA}{dx} = 2x$

$dA = 2 \cdot x \cdot dx$   
 $2(10)(\pm \frac{1}{5}) \rightarrow dA = \pm 4 \text{ cm}^2$

Actual  $A(10 + \frac{1}{5}) =$   
 Change

Taylor Polynomials  
 $f(x) = \sin(x)$ , center at 0.  
 $P_0(x) = f(0) = 0$   
 $P_0(x) = 0$     $T_0(x) = 0$   
 $P_1(x) = f(0) + \frac{f'(0)(x-0)}{1!}$

Suppose we have a function  $f(x)$  that we can differentiate as many times as we can, then the Taylor polynomial of order  $n$  generated by  $f$  at  $x = a$  is

Taylor Polynomial (a.k.a Taylor Series):  
 (generated by  $f$  at  $x = a$ )

$$P(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots$$

$P_2(x) = f(0) + f'(0)(x-0) + \frac{f''(0)(x-0)^2}{2!} = x$

for  $f(x) = \sin x$   
 center at  $x=0$     $P_1(x)$

$P_3(x) = f(0) + f'(0)(x-0) + \frac{f''(0)(x-0)^2}{2!} + \frac{f'''(0)(x-0)^3}{3!}$



Find  $\frac{dy}{dx}$ ?

$$y = \sin(\cos(x^2 - 3x))$$

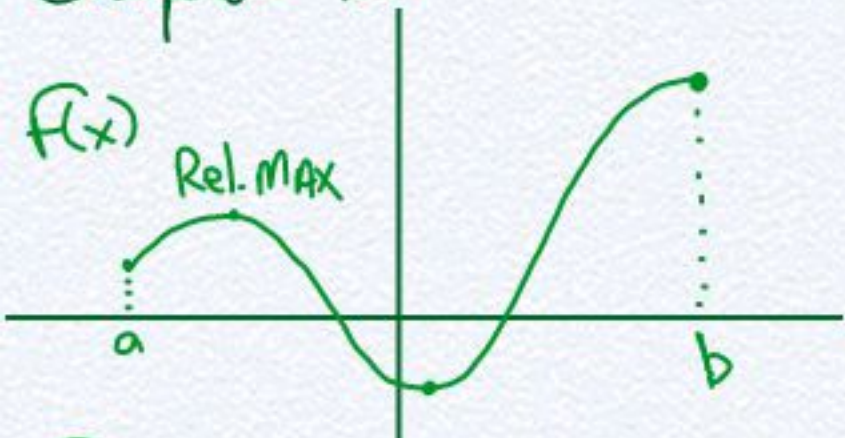
$$\frac{dy}{dx} = \cos(\cos(x^2 - 3x)) \cdot (-\sin(x^2 - 3x)) \cdot (2x - 3)$$

$$\frac{dy}{dx} y = e^{\sin x} \rightarrow e^{\sin x} (\cos x)$$

$$\frac{dy}{dx} y = \frac{5^{x^2-1}}{x^5-1} \rightarrow \frac{[5^{x^2-1} \cdot \ln 5 (2x) \cdot x^5 - 1] - [(5x^4 \cdot 5^{x^2-1})]}{(x^5-1)^2}$$

$$\frac{dy}{dx} y = \frac{\sqrt[5]{x^2-1} (7x^2-3x+1)^5}{x^4(x+1)}$$

## Chapter 4.2



$$f(x) = 6x^5 + 33x^4 - 30x^3 + 100$$

find the critical points:

$$\frac{d}{dx} f'(x) = 30x^4 + 132x^3 - 90x^2$$

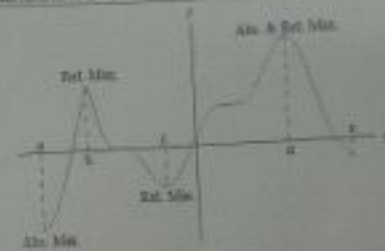
$$f'(x) = x^2(30x^2 + 132x - 90)$$

$$x^2 = 0 \quad 30x^2 + 132x - 90 = 0$$

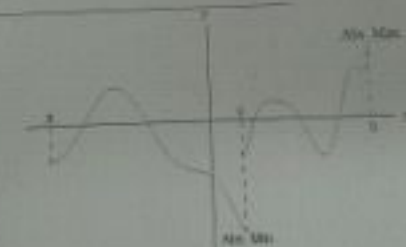
$$x = 0, x = \frac{3}{5}, x = -5$$

- Definition
- We say that  $f(c)$  has an absolute (or global) maximum at  $x=c$  if  $f(x) \leq f(c)$  for every  $x$  in the domain we are working on.
  - We say that  $f(c)$  has a relative (or local) maximum at  $x=c$  if  $f(x) \leq f(c)$  for every  $x$  in some open interval around  $x=c$ .
  - We say that  $f(c)$  has an absolute (or global) minimum at  $x=c$  if  $f(x) \geq f(c)$  for every  $x$  in the domain we are working on.
  - We say that  $f(c)$  has a relative (or local) minimum at  $x=c$  if  $f(x) \geq f(c)$  for every  $x$  in some open interval around  $x=c$ .

The maximum and minimum points of a function form a collection called the Extrema of the function.



Extreme Value Theorem  
Suppose that  $f(x)$  is continuous on the interval  $[a, b]$ . Then there are two numbers  $a \leq c, d \leq b$  such that  $f(c)$  is an absolute maximum for the function and  $f(d)$  is an absolute minimum for the function.



The graph is discontinuous at  $x=c$ , yet it has an absolute min at  $x=c$ .

Fermat's Theorem  
If  $f(x)$  has a relative extrema at  $x=c$  and  $f'(c)$  exists then  $x=c$  is a critical point of  $f(x)$ . In fact, it will be a critical point such that  $f'(c) = 0$ .

What is a critical point?  
How do you find it?  
How do you find the Absolute Max & Min on a closed interval?

## finding Extrema on a closed Interval:

Given  $f(x)$  on  $[a, b]$

Step 1: Find  $f'(x)$

Step 2: Find the critical points.

$$f'(x) = 0 \text{ or } f'(x) = \text{undefined.}$$

Step 3: Evaluate the function at the End points

$$x = a \quad f(a)$$

$$x = b \quad f(b)$$

at the critical points

$$x = c \quad f(c)$$

$$x = d \quad f(d)$$

$$f(x) = \sqrt[5]{x^2 - 6x} = (x^2 - 6x)^{\frac{1}{5}}$$

$$f'(x) = \frac{1(2x-6)}{5(x^2-6x)^{\frac{4}{5}}}$$

$$f'(x) = \frac{2x-6}{5 \cdot \sqrt[5]{(x^2-6x)^4}}$$

$$f'(x) = 0 \quad 2x - 6 = 0 \quad x = 3$$

$$f'(x) = \text{undefined} \quad x^2 - 6x = 0 \quad x = 0, x = 6$$

HW 19 Nov 12

1. finish 4.2
  2. Read 4.3
  3. Project #2 Syllabus
- Chapter 4.4



# Applications of Derivatives

## Chapter 4.2 Extrema on an Interval


### • Extreme Value Theorem

• A function may not have Extrema  
 • critical values

### • Fermat's Theorem

• Concave up/down  
 • Inflection points

$F'(c) = 0$   
 $F''(c) < 0$



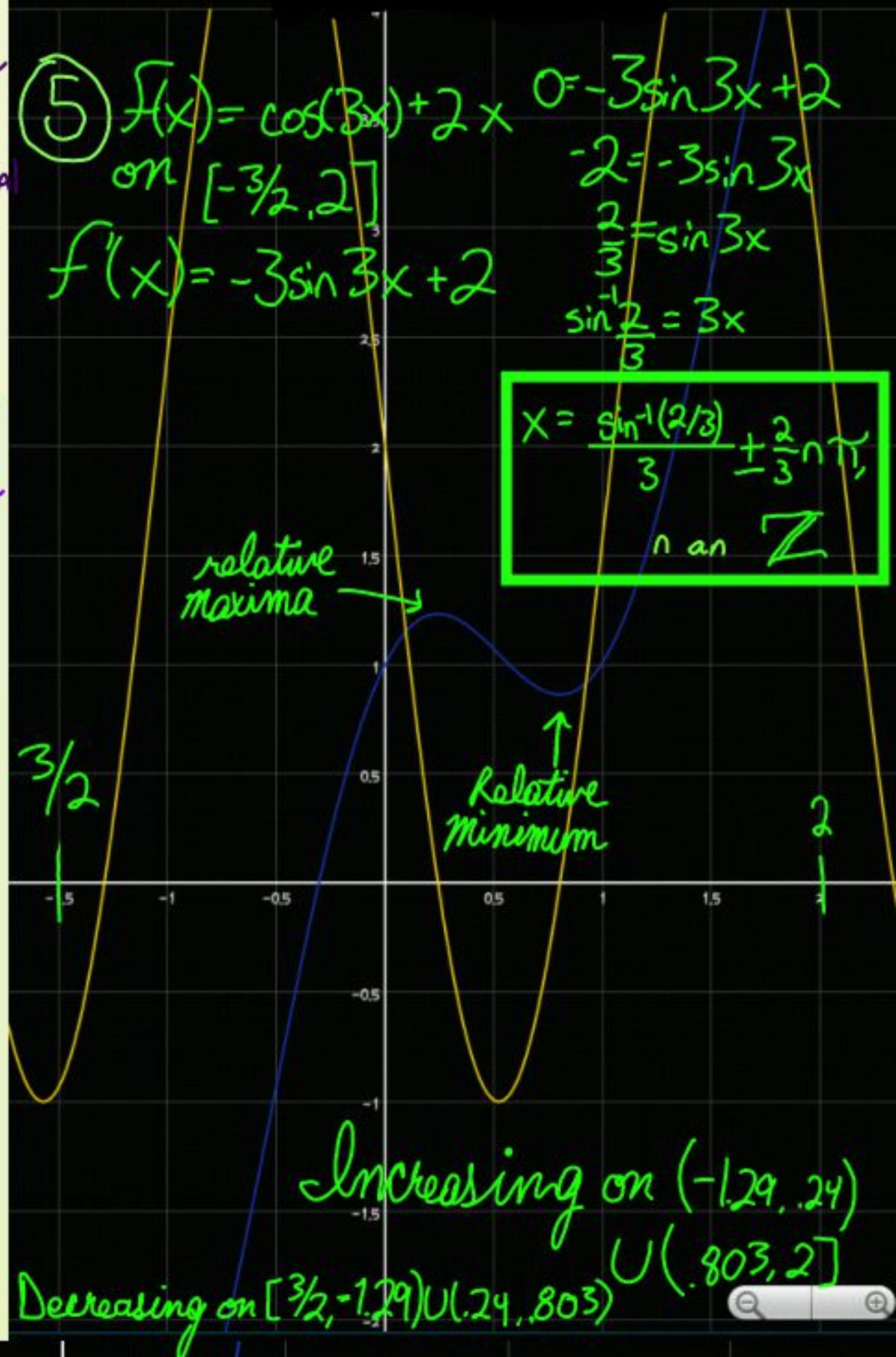
$F'(c) = 0$   
 $F''(c) > 0$



⑤  $f(x) = \cos(3x) + 2x$   
 on  $[-\frac{3}{2}, 2]$   
 $f'(x) = -3\sin 3x + 2$

$0 = -3\sin 3x + 2$   
 $-2 = -3\sin 3x$   
 $\frac{2}{3} = \sin 3x$   
 $\sin^{-1} \frac{2}{3} = 3x$

$x = \frac{\sin^{-1}(2/3)}{3} + \frac{2}{3}n\pi,$   
 $n \text{ an } \mathbb{Z}$



Increasing on  $(-1.29, .24)$   
 $\cup (.803, 2]$

Decreasing on  $[\frac{3}{2}, -1.29) \cup (.24, .803)$

⑫  $f(x) = -x^3 + 6x^2 + 12$   
 $f'(x) = -3x^2 + 12x$   
 $f''(x) = -6x + 12$

Homework: 4.3 WorkSheet  
 #6, 14, 15, 16-19

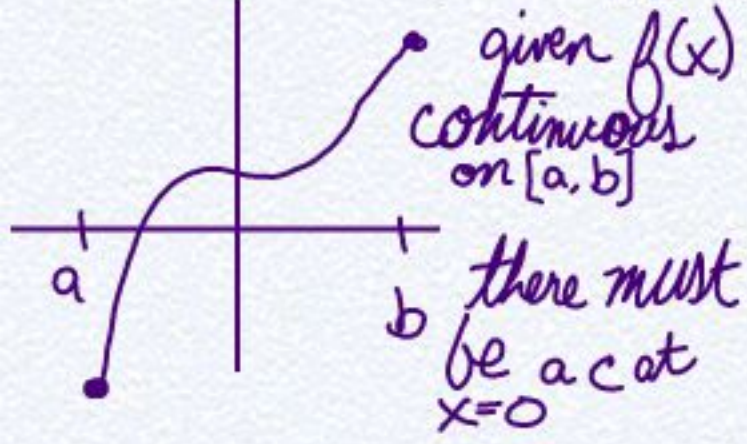




# Must Know!

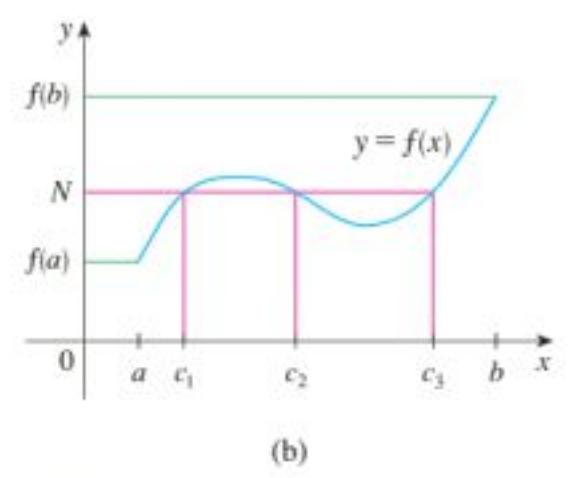
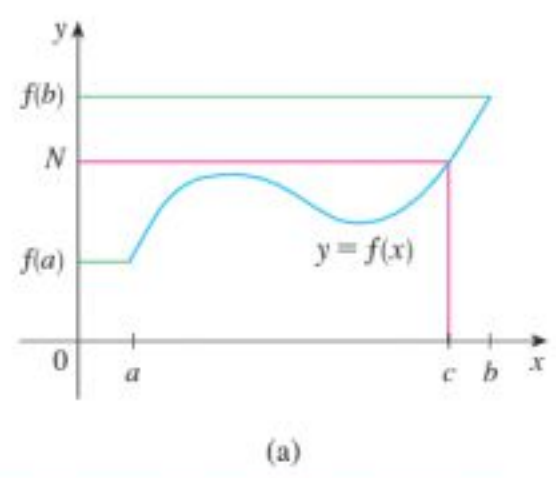
## Chapter 2.4 page 120

### Intermediate value Theorem



**10 The Intermediate Value Theorem** Suppose that  $f$  is continuous on the closed interval  $[a, b]$  and let  $N$  be any number between  $f(a)$  and  $f(b)$ , where  $f(a) \neq f(b)$ . Then there exists a number  $c$  in  $(a, b)$  such that  $f(c) = N$ .

The Intermediate Value Theorem states that a continuous function takes on every intermediate value between the function values  $f(a)$  and  $f(b)$ . It is illustrated by Figure 8. Note that the value  $N$  can be taken on once [as in part (a)] or more than once [as in part (b)].

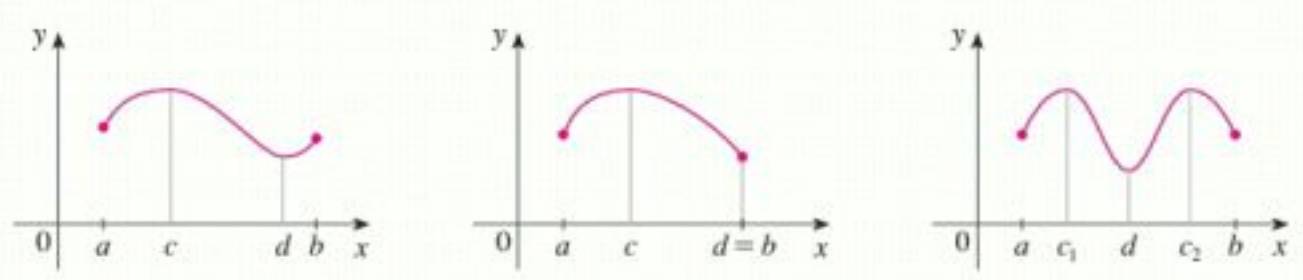


## Chapter 4.2 page 264

### Extreme Value Theorem

**3 The Extreme Value Theorem** If  $f$  is continuous on a closed interval  $[a, b]$ , then  $f$  attains an absolute maximum value  $f(c)$  and an absolute minimum value  $f(d)$  at some numbers  $c$  and  $d$  in  $[a, b]$ .

The Extreme Value Theorem is illustrated in Figure 7. Note that an extreme value can be taken on more than once. Although the Extreme Value Theorem is intuitively very plausible, it is difficult to prove and so we omit the proof.



## Chapter 4.2 Fermat's Theorem:

**4 Fermat's Theorem** If  $f$  has a local maximum or minimum at  $c$ , and if  $f'(c)$  exists, then  $f'(c) = 0$ .

## Chapter 4.3 Mean Value Theorem:

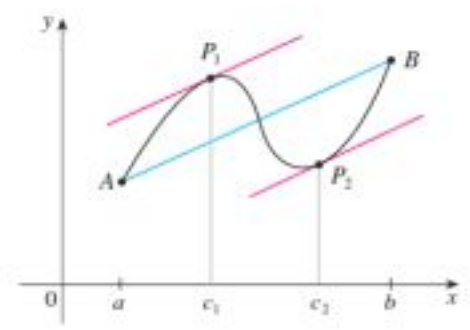
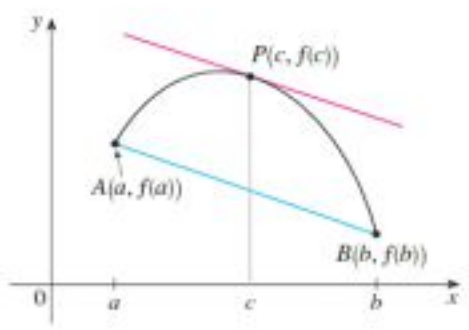
**The Mean Value Theorem** If  $f$  is a differentiable function on the interval  $[a, b]$ , then there exists a number  $c$  between  $a$  and  $b$  such that

**1** 
$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

or, equivalently,

**2** 
$$f(b) - f(a) = f'(c)(b - a)$$

We can see that this theorem is reasonable by interpreting it geometrically. Figures 1 and 2 show the points  $A(a, f(a))$  and  $B(b, f(b))$  on the graphs of two differentiable functions.



The slope of the secant line  $AB$  is

$$m_{AB} = \frac{f(b) - f(a)}{b - a}$$



$f(x) = 2x^3 - x^2$  on  $[0, 2]$   
 find  $c$  such that  
 $\frac{f(b) - f(a)}{b - a} = f'(c)$

Step 1:  $f(a) = f(0) = 0$   
 $f(b) = f(2) = 12$

Step 2:  $f'(x) = 6x^2 - 2x$

Step 3:  $f'(c) = 6c^2 - 2c$

$\frac{f(b) - f(a)}{b - a} = 6 \therefore 6 = 6c^2 - 2c$

$0 = 6c^2 - 2c - 6$

Quadratic  $\rightarrow$

$c = \frac{1 \pm \sqrt{37}}{6}$

$V = \frac{4}{3} \pi r^3 \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

11  $f(x) = 8x + e^{-3x}$  on  $[2, 3]$

$f(-2) = 8(-2) + e^{-3(-2)}$   
 $-16 + e^6$

$f(3) = 8(3) + e^{-3(3)}$   
 $24 + e^{-9}$

$\frac{f(b) - f(a)}{b - a} = \frac{24 + e^{-9} - (-16 + e^6)}{3 - (-2)}$

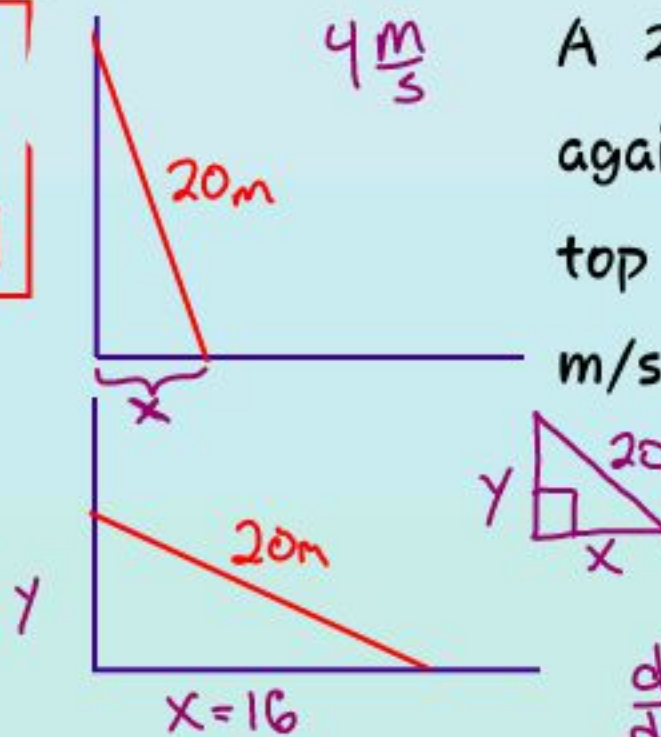
$= \frac{24 + e^{-9} + 16 - e^6}{5}$

$= \frac{40 - e^6 + e^{-9}}{5} = -72.686$

$\therefore f'(x) = 8 - 3e^{-3x}$

$f'(c) = 8 - 3e^{-3c} = -72.686$

4.2 syllabus  
 4.3 syllabus  
 2.8, 1.10, 1.15, 1.19



A 20 m ladder leans against a wall and the top slides down at 4 m/s

$x^2 + y^2 = 20^2$

$\frac{dx}{dy} \{x^2 + y^2 = 20^2\}$

$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$

**Truck Problem:**  
 Truck A travels east at 40 mi/hr.  
 Truck B travels north at 30 mi/hr.  
 How fast is the distance between the trucks changing 6 minutes later?  
 Given:  $\frac{dx}{dt} = 40$  mi/hr  
 $\frac{dy}{dt} = 30$  mi/hr  
 $t = 6$  minutes  
 Find:  $\frac{dz}{dt} = ???$

**TRUCK Problem**

$\frac{dz}{dt} = ???$   
 $t = 6 \text{ min} = \frac{1 \text{ hr}}{10}$

Given:  $\frac{dx}{dt} = 40$   
 $\frac{dy}{dt} = 30$

Formula:  $x^2 + y^2 = z^2$   
 $2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$   
 $x \frac{dx}{dt} + y \frac{dy}{dt} = z \frac{dz}{dt}$

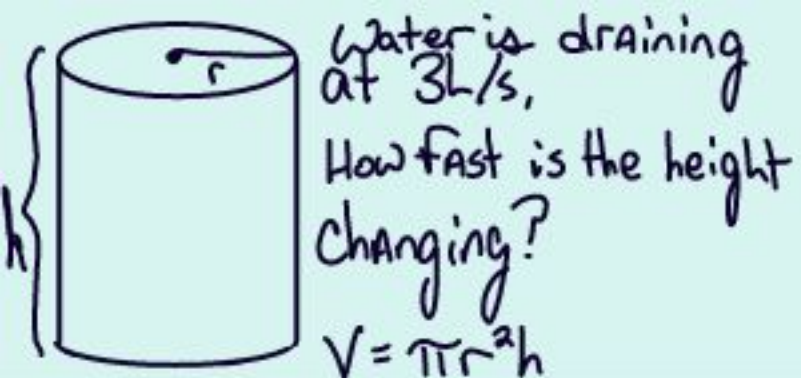
Find  $\frac{dz}{dt} = ?$

**The sliding ladder**  
 A 20 m ladder leans against a wall. The top slides down at a rate of 4 m/sec. How fast is the bottom of the ladder moving when it is 16 m from the wall?  
 Find:  $\frac{dx}{dt} = ???$

Given:  $x = 16$   
 $\frac{dy}{dt} = -4$  m/sec

$x^2 + y^2 = 20^2$   
 $2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$   
 $\frac{dx}{dt} = -\frac{y}{x} \frac{dy}{dt}$   
 $\frac{dx}{dt} = -\frac{12}{16} \cdot (-4) = 3$





Water is draining at 3L/s,  
How fast is the height changing?

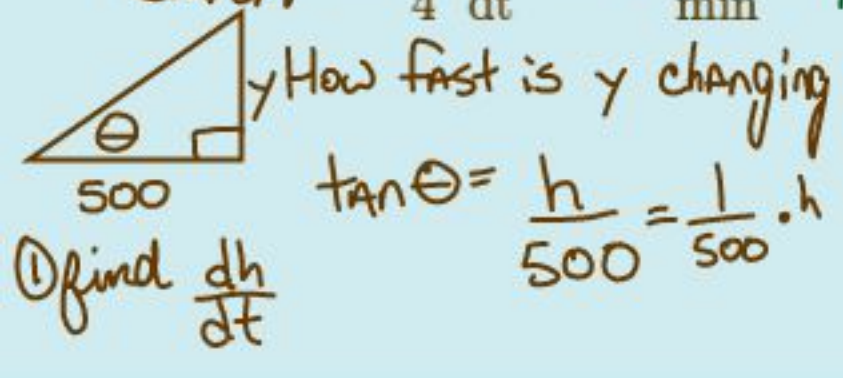
$$V = \pi r^2 h$$

$$1 \text{ L} = 1000 \text{ cm}^3 \quad \frac{dV}{dt} = -3 \text{ L/s} = -3000 \frac{\text{cm}^3}{\text{s}}$$

$$-3000 \frac{\text{cm}^3}{\text{s}} = \pi r^2 \frac{dh}{dt}$$

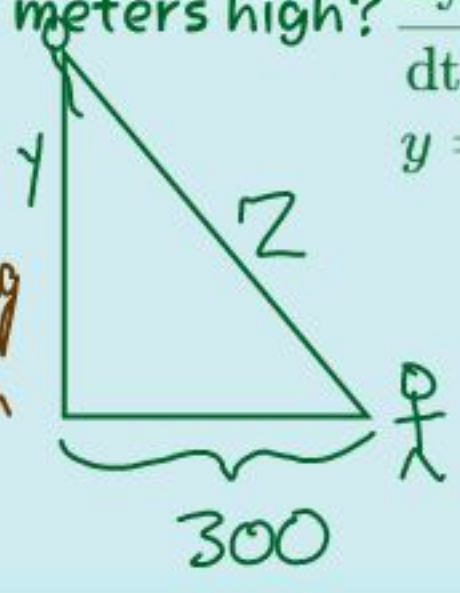
1-10  
WS  
4.2  
4.3

Given  $\theta = \frac{\pi}{4} \frac{d\theta}{dt} = .14 \frac{\text{rad}}{\text{min}}$



$$\tan \theta = \frac{h}{500} = \frac{1}{500} \cdot h$$

A weather balloon is rising vertically at the rate of 5 meters per second. An observer is standing on the ground 300 meters from the point where the balloon was released. At what rate is the distance between the observer and the balloon changing when the balloon is 400 meters high?



$$\frac{dy}{dt} = \frac{+5 \text{ m}}{\text{second}}$$

$$y = 400 \text{ m}$$

Formula:

$$x^2 + y^2 = z^2$$

$$300^2 + y^2 = z^2$$

$$\frac{dy}{dt} \Rightarrow 90,000 + y^2 = z^2$$

$$0 + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$$

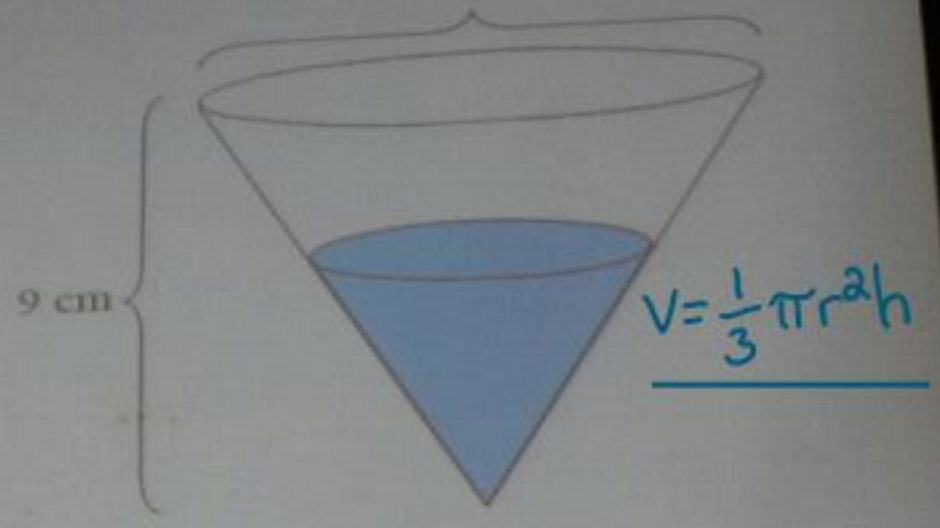
$$y \frac{dy}{dt} = z \cdot \frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{y}{z} \frac{dy}{dt}$$

$$\frac{dz}{dt} = \frac{400}{500} \cdot 5 = \boxed{4 \frac{\text{m}}{\text{s}}}$$

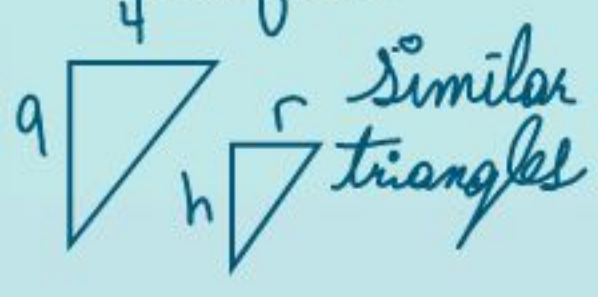
### Draining a tank

A cone filter of diameter 8 cm and height 9 cm is draining at a rate of 2 cm<sup>3</sup>/min. Find the rate at which the fluid depth h decreases when h = 5 cm.



cylinder } Volume = area x height  
=  $\pi r^2 h$

Surface Area = Top + Bottom + side



Similar triangles

$$\frac{9}{h} = \frac{4}{r}$$

$$9r = 4h$$

$$r = \frac{4}{9} h$$

$$\frac{dV}{dt} = \frac{-2 \text{ cm}^3}{\text{min}} \quad \text{find } \frac{dh}{dt}$$

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi \left(\frac{4}{9} h\right)^2 \cdot h$$

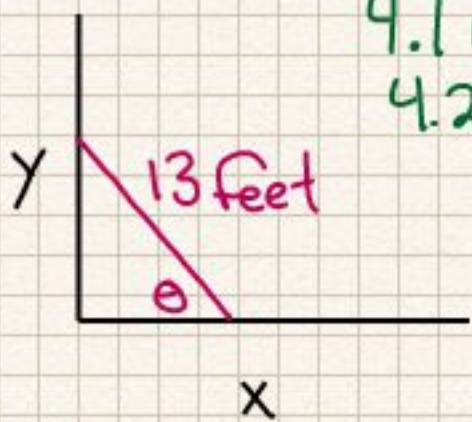
$$V = \frac{16}{3 \cdot 81} \pi h^3 \Rightarrow \frac{dV}{dt} = \frac{16}{81} \pi h^2 \frac{dh}{dt} = \frac{-2 \text{ cm}^3}{\text{min}}$$

$$\frac{-2 \text{ cm}^3}{\frac{16}{81} \pi h^2} = \frac{dh}{dt}$$



4.1 (6)

Homework:  
4.1 Syllabus  
4.1 WS  
4.2/3 WS



$\frac{dx}{dt} = \frac{3 \text{ feet}}{\text{min}}$  find  $\frac{d\theta}{dt}$   
 $x = 12 \text{ feet}$

$\cos \theta = \frac{x}{13} \Rightarrow -\sin \theta = \frac{1}{13} \cdot \frac{dx}{dt}$

$\frac{d\theta}{dt} = -\frac{1}{13 \cdot \sin \theta} \cdot \frac{dx}{dt}$   
 $= \frac{-1}{13 \cdot \frac{5}{13}} \cdot 3$

### 4.5 Optimization Problems

2 or more variables  
↳ use substitution

$2l + 2w = 300$   
 $w = 150 - l$

$A = lw = l(-l + 150)$   
 $-l^2 + 150l$

$\frac{-b}{2a} = \frac{-150}{-2} = 75$

### 4.6 Worksheet

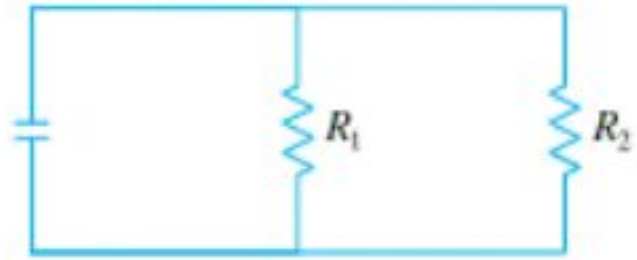
1-11  
4.2/3 finish WS  
4.1 Worksheet  
4.1 Syllabus 11/24/2012

Read 4.6 + try 1-11

35. If two resistors with resistances  $R_1$  and  $R_2$  are connected in parallel, as in the figure, then the total resistance  $R$ , measured in ohms ( $\Omega$ ), is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

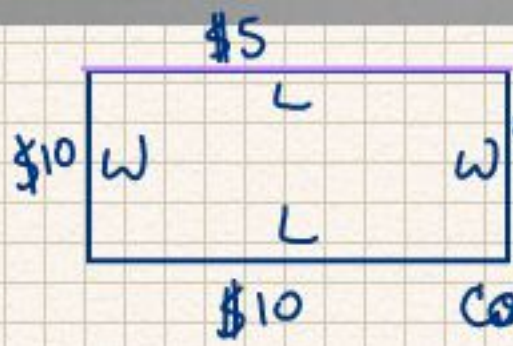
If  $R_1$  and  $R_2$  are increasing at rates of  $0.3 \Omega/s$  and  $0.2 \Omega/s$ , respectively, how fast is  $R$  changing when  $R_1 = 80 \Omega$  and  $R_2 = 100 \Omega$ ?



Given  $= R^{-1} = R_1^{-1} + R_2^{-1}$   
 $\Rightarrow -R^{-2} \frac{dR}{dt} = -R_1^{-2} \left(\frac{dR_1}{dt}\right) - R_2^{-2} \left(\frac{dR_2}{dt}\right)$   
 $\Rightarrow \frac{1}{R^2} \frac{dR}{dt} = \frac{1}{R_1^2} \left(\frac{dR_1}{dt}\right) + \frac{1}{R_2^2} \left(\frac{dR_2}{dt}\right)$

$\frac{dR}{dt} = R^2 \left[ \frac{1}{R_1^2} \left(\frac{dR_1}{dt}\right) + \frac{1}{R_2^2} \left(\frac{dR_2}{dt}\right) \right]$

**Example 2: Minimizing Cost**  
A rectangular garden of area 75 square feet is to be surrounded on three sides by a brick wall costing \$10 per foot and on one side by a fence costing \$5 per foot. Find the dimensions of the garden such that the cost of materials is minimized.

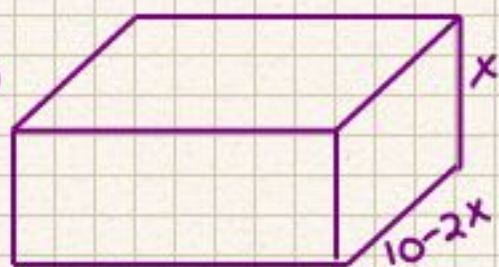
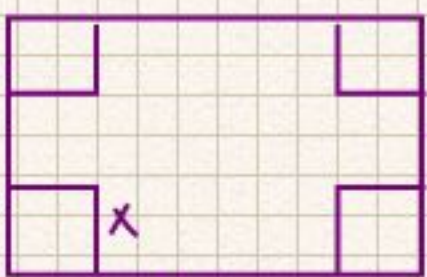


$A = 75 \text{ m}^2 \quad l = \frac{75}{w}$   
 $lw = 75 \text{ m}^2$

$\text{Cost} = 10w + 10w + 10l + 5l$   
 $C = 20w + 15l$   
 $C = 20w + 15\left(\frac{75}{w}\right)$

$\frac{dC}{dw} = 20 - \frac{15 \cdot 75}{w^2}$   
 $20 = \frac{1125}{w^2} \quad w^2 = \frac{1125}{20}$   
 $w = \sqrt{\frac{1125}{20}} \quad w = 7.5 \text{ m}$





$$l = 16 - 2x \quad 0 \leq x \leq 5 \text{ feasible domain}$$

$$w = 10 - 2x \quad 5 \quad V = lwh$$

$$h = x \quad V = (16 - 2x)(10 - 2x)x$$

$$V = 4x^3 - 52x^2 + 160x$$

$$V' \Rightarrow 12x^2 - 104x + 160$$

$$-160 < 12x^2 - 104x$$

$$-40 < 3x^2 - 26x$$

Four Feet of wire is used to form a square and a circle.

Let  $x$  = wire used for  $\square$

$$\square \frac{x}{4} \quad \text{The remaining wire}$$

$$\frac{x}{4} = 4 - x$$

$$c = 4 - x$$



$$2\pi r = 4 - x$$

$$r = \frac{4 - x}{2\pi}$$

Goal: Maximize  $A(\square + \circ)$

$$A = A_{\square} + A_{\circ}$$

$$A = \left(\frac{x}{4}\right)^2 + \pi r^2$$

$$= \frac{x^2}{16} + \pi \left(\frac{4-x}{2\pi}\right)^2$$

$$= \frac{x^2}{16} + \frac{(4-x)^2}{4\pi} \quad 0 \leq x \leq 4$$

$$\frac{dA}{dx}$$

HW: Worksheet 4.6  
only 1-11, 13-16  
12, 17



$$V = lL$$

$$V = 1000 \text{ cm}^3$$

$$= \pi r^2 h = 1000$$

$$h = \frac{1000}{\pi r^2}$$

Minimize Surface Area

$$S = \text{Top} + \text{Bottom} + \text{side}$$

$$\pi r^2 + \pi r^2 + 2\pi r h$$

$$2\pi r^2 + 2\pi r h$$

$$2\pi r^2 + 2\pi r \left(\frac{1000}{\pi r^2}\right)$$

$$2\pi r^2 + 2\pi r \left(\frac{1000}{\pi r^2}\right)$$



# 4.8 Antiderivatives

Find  $f(x) = \text{given}$   
 $f'(x) = x^3 - 4x^2 + 1$

$$f(x) = \frac{x^{3+1}}{4} - \frac{4}{3}x^3 + 1x$$

anti-derivative

$$f'(x) = \frac{1}{x} \Rightarrow f(x) = \ln|x|$$

$$\lim_{x \rightarrow 9} \frac{\sqrt{x^2 - 3}}{x - 9}$$

Use L'Hôpital's

$$\lim_{\Delta \rightarrow 0} \left( \frac{\sin \Delta}{\Delta} \right) = 1$$

$$\lim_{x \rightarrow \infty} x \cdot \sin\left(\frac{1}{x}\right) =$$

$$y' = 2x \quad (x, y)$$

$$y = x^2 + C \quad (1, 4)$$

General Solution

$$4 = 1^2 + C \Rightarrow y = x^2 + 3$$

Specific Solution

Homework 4.8  
Syllabus 4.6W

This is called an initial value problem

$\infty$   $\frac{0}{0}$   $\infty - \infty$   
 $\frac{\infty}{\infty}$   $\infty \cdot 0$   $\infty^0$

$$3) \lim_{t \rightarrow \infty} \frac{\ln(3t)}{t^2} \quad \begin{matrix} \infty - \infty \text{ can be} \\ \frac{0}{0} \text{ or } \frac{\infty}{\infty} \end{matrix}$$

$$= \frac{\infty}{\infty}$$

L'Hôpital's  $\Rightarrow$   
 let  $f(t) = \ln(3t) \Rightarrow f'(t) = \frac{1}{3t} \cdot 3$   
 let  $g(t) = t^2 \Rightarrow g'(t) = 2t$

$$\frac{\frac{3}{3t}}{\frac{2t}{1}} = \frac{A}{B} = \frac{AD}{BC} = \frac{3}{6t^2} = 0$$

$$\lim_{x \rightarrow 0^+} \left( \frac{1}{x} - \frac{1}{\sin x} \right)$$

$$= \lim_{x \rightarrow 0^+} \left( \frac{\sin x}{x \sin x} - \frac{x}{x \sin x} \right)$$

Function	Particular antiderivative	Function	Particular antiderivative
$cf(x)$	$cF(x)$	$\sin x$	$-\cos x$
$f(x) + g(x)$	$F(x) + G(x)$	$\sec^2 x$	$\tan x$
$x^n \quad (n \neq -1)$	$\frac{x^{n+1}}{n+1}$	$\sec x \tan x$	$\sec x$
$1/x$	$\ln x $	$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x$
$e^x$	$e^x$	$\frac{1}{1+x^2}$	$\tan^{-1} x$
$\cos x$	$\sin x$		



$$\lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}}$$

$$\ln f(x) = \ln (1+x)^{1/x}$$

$$\ln f(x) = \frac{1}{x} \ln(1+x)$$

$$\ln f(x) = \frac{\ln(1+x)}{x}$$

$$e^{\ln f(x)} = e^{\frac{\ln(1+x)}{x}}$$

$$\lim_{x \rightarrow 0} f(x) = e^{\frac{\ln(1+x)}{x}}$$

Indeterminate Forms:

$$\lim_{x \rightarrow 0^+} (1+x)^{\frac{1}{x}} \leftarrow 1^\infty$$

$$\lim_{x \rightarrow 0^+} \frac{\ln(1+x)}{x} \leftarrow \frac{\infty}{\infty}$$

$$f(x) = (1+x)^{\frac{1}{x}}$$

$$\ln f(x) = \ln(1+x)^{\frac{1}{x}}$$

$$\ln f(x) = \frac{1}{x} \ln(1+x)$$

$$e^{\ln f(x)} = e^{\frac{\ln(1+x)}{x}}$$

$$\lim_{x \rightarrow 0^+} \frac{\ln(1+x)}{x}$$

$$\lim_{x \rightarrow 0^+} f(x) = e^{\lim_{x \rightarrow 0^+} \frac{\ln(1+x)}{x}}$$

$$= \lim_{x \rightarrow 0^+} \frac{1/(1+x)}{1}$$

$$= \lim_{x \rightarrow 0^+} \frac{1}{1+x}$$

$$= 1$$

$$\lim_{x \rightarrow 0^+} (1+x)^{\frac{1}{x}} = e^1 = e$$

Indeterminate Products

$$\lim_{x \rightarrow 0} \left( x \sin \frac{1}{x} \right) \quad \text{This approaches } \infty \cdot 0$$

Rewrite!

$$\lim_{x \rightarrow 0} \frac{\sin \frac{1}{x}}{\frac{1}{x}} \quad \text{This approaches } \frac{0}{0}$$

We already know that  $\lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \right) = 1$   
but if we want to use L'Hôpital's rule:

$$\lim_{x \rightarrow 0} \frac{\sin \frac{1}{x}}{\frac{1}{x}}$$

$$= \lim_{x \rightarrow 0} \frac{\cos\left(\frac{1}{x}\right) \cdot \left(-\frac{1}{x^2}\right)}{-\frac{1}{x^2}} = \lim_{x \rightarrow 0} \cos\left(\frac{1}{x}\right) = \cos(0)$$

$$\lim_{x \rightarrow \infty} x^{1/x} \rightarrow \infty^0$$

$$f(x) = x^{1/x}$$

$$\ln f(x) = \frac{1}{x} \ln x$$

$$e^{\ln f(x)} = e^{\frac{\ln x}{x}}$$

$$f(x) = e^{\frac{\ln x}{x}}$$