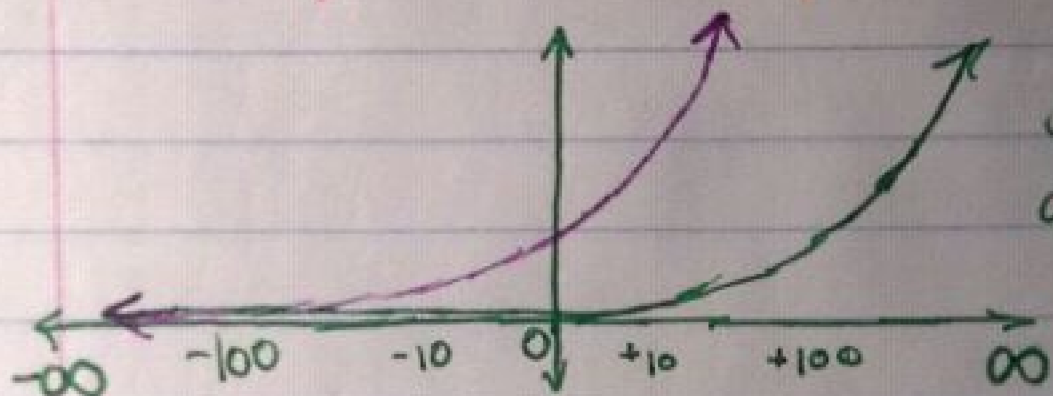
Exponential function Review

$f(x) = a(b)^x$

a: y-int, b: base multiplier, $b > \emptyset$, $b \neq 1$



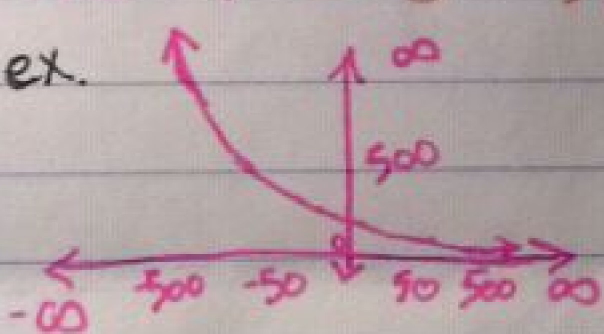
as $x \to -\infty$, $y \to \emptyset$ ex $y = 2^x$

as $x \to \infty$, $y \to \infty$ $y = 10^x$

$y = e^x$

When the base multiplier b is a number between $\emptyset$ and 1 $(0 < b < 1)$, it will be a decreasing exponential

ex.



as $x \to -\infty$, $y \to \infty$

as $x \to \infty$, $y \to \emptyset$

Change of bases

$\log_\Delta M = \frac{\log M}{\log \Delta}$

where $\Delta \neq 10$, $\Delta \neq e$

$2^x = 5$

$\log 2^x = \log 5$

$x \log 2 = \log 5$

$x = \frac{\log 5}{\log 2}$

$x \approx 2.322$

Product Rule of Logs

$\text{Log}_b(xyz) = \log_b x + \log_b y + \log_b z$

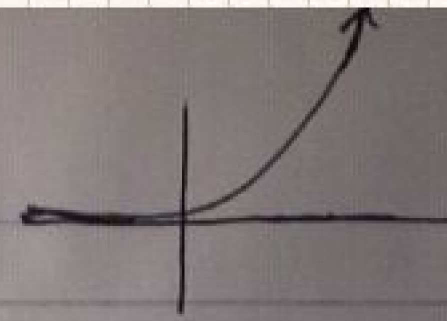
Quotient Rule

$\text{Log}_b \frac{x}{z} = \log_b x - \log_b z$

Power Rule of Log

$\text{Log}_b x^z = z \log_b x$

$f(x) = \frac{4x}{e^{\cos x}}$



One to One functions pass both the vertical line and horizontal line tests.

Given that a function $f(x)$ that is one-to-one, it has an inverse function, $f^{-1}(x)$

1) $f \circ f^{-1}(x) = x$
2) To find a function's inverse, swap x for y and solve for y.
3) The graph of a function and it's inverse are ~~sym~~ symmetrical with respect to origin.

Given $f(x) = 3x - 5$, find $f^{-1}(x)$

1) Rewrite $f(x)$ as $y=$
2) solve for x
3) swap x + y
4) Rewrite $y=$ as $f^{-1}(x)$

$2\log(2x+1) + \log x - 4\log$

• Vertical, Horizontal, diagonal Asymptote

if $f(x) = b^x$, where $b > 0$, $b \neq 1$, its inverse is $f^{-1}(x) = \log_b x$

if $\log_b 1 = \emptyset$ if $\log_b b = 1$ if $b^{\log_b x} = x$

$f(x) = \log_{10}(2x+3)$

$y = \log_{10}(2x+3)$

$10^y = 2x+3$

$10^y - 3 = 2x$

$\frac{10^y - 3}{2} = x$ $\frac{10^x - 3}{2} = f^{-1}(x)$

$\log_5 (x-5)(x+4)^2$

$\log_5(x-5) + \log_5(x+4) + \log_5(x+4)$

$\log_5(x-5) + 2\log_5(x+4)$

27) $f(x) = \dfrac{x^2}{1+2x}$

$f'(x) = \dfrac{2x^2+2x}{(1+2x)^2}$

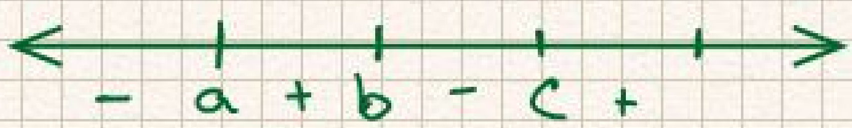
$f''(x) = (4x+2)(1+2x)^2 - (2x^2+2x)(4+8x) \div (1+2x)^4$

$= 2(2x+1)(1+2x)^2 - (2(x^2+x)4(1+2x$

$= \dfrac{2(2x+1)}{(1+2x)^4} = \dfrac{2}{(2x+1)^3}$

[redacted]

$f'(x)$



Given $f(x)$ is differentiable, on the interval $f'(x)>0$, $f(x)$ is increasing. If $f'(x)<0$, $f(x)$ is decreasing. Where $f'(x)=0$, $f(x)$ will have a max or a minimum. On the interval $f''(x)>0$, $f(x)$ is concave up. If $f''(x)<0$, $f(x)$ is concave down.

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1) $\lim\limits_{z\to 0} \dfrac{\sin(10z)}{z} = \underset{1\cdot 10}{10}$

$\lim\limits_{\Delta\to 0} \dfrac{\sin\Delta}{\Delta} = 1$

2) $\lim\limits_{\alpha\to 0} \dfrac{\sin(12\alpha)}{\sin(5\alpha)} = \dfrac{12}{5}$

3) $\lim\limits_{x\to 0} \dfrac{\cos(4x)-1}{x} \cdot 4 = 0\cdot 4 = 0$

$=$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

given $f(x) = \sin x$, find $f'(x)$

$f(x+h) = \sin(x+h)$ $f(x) = \sin x$

$f(x+h) - f(x)$

$= \sin x\cos h + \cos x\sin h - \sin x$

Given $f(x) = \tan x$.
find $f'(x)$

$\tan x = \dfrac{\sin x}{\cos x}$

let $g(x) = \sin x,\ g'(x) = \cos x$

let $h(x) = \cos x,\ h'(x) = -\sin x$ $\therefore \dfrac{\cos x(\cos x) - [\sin x(-\sin x)]}{(\cos x)^2}$

$= \dfrac{[\cos^2 x + \sin^2 x]}{\cos^2 x} = \dfrac{1}{\cos^2 x} \Rightarrow \boxed{\sec^2 x} = f'(\tan x)$

$f(x) = \cot(x)$

$\cot(x) = \dfrac{\cos(x)}{\sin x}$

$f(x) = \cos(x) \rightarrow f'(x) = -\sin(x)$ $f(x) = \tan(x) \rightarrow f'(x) = \sec^2(x)$

$g(x) = \sin(x) \rightarrow g'(x) = \cos(x)$ $f(x) = \cot(x) \rightarrow f'(x) = -\csc^2(x)$

$\dfrac{\sin(x)(-\sin(x)) - [\cos(x)(\cos(x))]}{\sin^2(x)}$

$= \dfrac{-\sin^2 x - \cos^2 x}{\sin^2 x} \rightarrow \dfrac{-1(\sin^2 x + \cos^2 x)}{\sin^2(x)} \rightarrow \dfrac{-1}{\sin^2(x)} \rightarrow \boxed{-\csc^2(x)}$

$f(x) = \sec x = \dfrac{1}{\cos x} \rightarrow \dfrac{[\cos x(0) - (1)(-\sin x)]}{\cos^2(x)} = \dfrac{\tan(x)}{\cos(x)} = \boxed{\tan(x)\sec(x)}$

let $f(x) = 1,\ f'(x) = 0$

let $g(x) = \cos x,\ g'(x) = -\sin x$

$f(x) = \csc(x) = \dfrac{1}{\sin x} = \dfrac{-\cos(x)}{\sin^2(x)}$ $\dfrac{-\cos(x)}{\sin(x)} \cdot \dfrac{1}{\sin(x)} = \boxed{-\cot(x)\csc(x)}$

$f(x) = 1,\ f'(x) = 0$

$g(x) = \sin(x),\ g'(x) = \cos(x)$

Given $f(x)$ & $g(x)$, find $f[g(x)]$

① $f(x) = \sqrt{x},\ g(x) = 2x-1$

$f \circ g(x) = \sqrt{2x-1}$

② $f(x) = \sin(x),\ g(x) = \sqrt{x}$

$f \circ g(x) = \sin\sqrt{x}$

③ $f(x) = e^x$ $g(x) = \tan(x)$

$f \circ g(x) = e^{\tan(x)}$

Given

$F = (f \circ g)(x) = f[g(x)]$

then $\dfrac{dF}{dx} = [f'(x)g(x)] \cdot g'(x)$

$F(x) = \sqrt{\tan x} = \tan x^{1/2}$

let $f(x) = x^{1/2}$, $g(x) = \tan x$

$f'(x) = \dfrac{1}{2\sqrt{x}}$ $\quad g'(x) = \sec^2 x$

$f'[g(x)] \cdot g'(x)$ or $\frac{1}{2}\tan x^{-1/2} \cdot \sec^2 x$

$\dfrac{1}{2\sqrt{\tan x}} \cdot \sec^2 x$

$F(x) = (2x^2 - 5x + 10)$

let $f(x) = x^{99}$, $f'(x) = 99x^{98}$ $\longrightarrow$ $99(2x^2-5x+10)^{98} \cdot (4x-5)$

let $g(x) = 2x^2 - 5x + 10$, $g'(x) = 4x - 5$

$F(x) = \sqrt{x^2+1}$

let $f(x) = \sqrt{x}$, $f'(x) \dfrac{1}{2\sqrt{x}}$

let $g(x) = x^2+1$, $g'(x) = 2x$

$f'g(x) \cdot g'(x) \to \dfrac{1}{2\sqrt{x^2+1}} \cdot 2x = \dfrac{x}{\sqrt{x^2+1}} = \boxed{\dfrac{x\sqrt{x^2+1}}{x^2+1}}$

$F(x) = \sin(x^2)$

$f(x) = \sin(x) \to f'(x) = \cos(x)$ 3.3

$g(x) = x^2 \to g'(x) = 2x$

$\cos(x^2) \cdot 2x = 2x\cos(x^2)$

$F(x) = \sin^2(x) = [\sin x]^2$

$f(x) = x^2$ $\quad f'(x) = 2x$

$g(x) = \sin(x)$ $\quad g'(x) = \cos(x)$

$2(\sin x) \cdot \cos x$

$= \sin(2x)$ ☆ work on trig

8 $(4x - x^2)^{100}$

$F(x) = x^{100}$ $\quad F'(x) = 99$

$g(x) = 4x - x^2$ $\quad g'(x) = -2x + 4$

$100(4x-x^2)^{99} \cdot (-2x+4)$

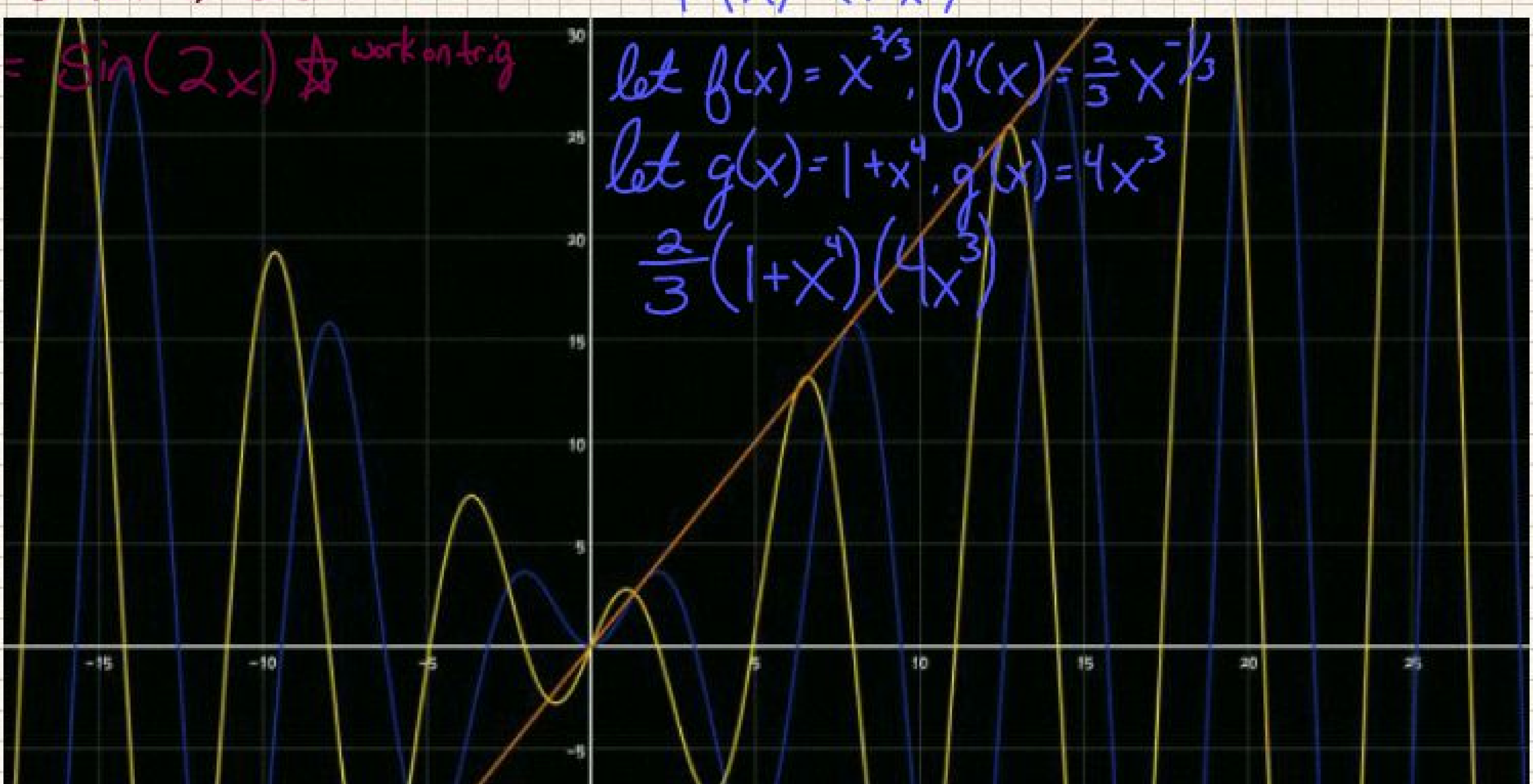
$= 200(-x+2)(4x-x^2)^{99}$

10 $F(x) = (1+x^4)^{2/3}$

let $f(x) = x^{2/3}$, $f'(x) = \frac{2}{3}x^{-1/3}$

let $g(x) = 1 + x^4$, $g'(x) = 4x^3$

$\frac{2}{3}(1+x^4)(4x^3)$



If $F(x) = \sin(x^2-4)$

let $f(x) = \sin x$

let $g(x) = x^2 - 4$

$\frac{d}{dx} f(g(x)) = f'(g(x))(g'(x))$

$\frac{d}{dx}\left[\sqrt[3]{\cos(e^{\sec x})}\right]$

$= \frac{d}{dx} \cos(e^{\sec x})^{1/3}$

$= \frac{-1}{3}\left[\cos(e^{\sec x})\right]^{-2/3} \sin(e^{\sec x})(e^{\sec x})(\sec x \tan x)$

$$y = \sqrt{\sin(e^{\cos x})}$$

$= \left[\sin(e^{\cos x})\right]^{1/2}$

$= \boxed{\frac{1}{2\sqrt{\sin(e^{\cos x})}} \cdot (\cos(e^{\cos x}).\ e^{x}(-\sin x)}$

Understand the problem

Come up with a plan

Execute the plan

Check your work

For what values of r does $y = e^{rt}$ satisfy the differential equation

$y'' - 5y' + 6y = 0$

$y = e^{rt} \quad y' = re^{rt} \quad y'' = r(re^{rt})$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

$x = \frac{1}{2}t \quad y = t^2 - 3$

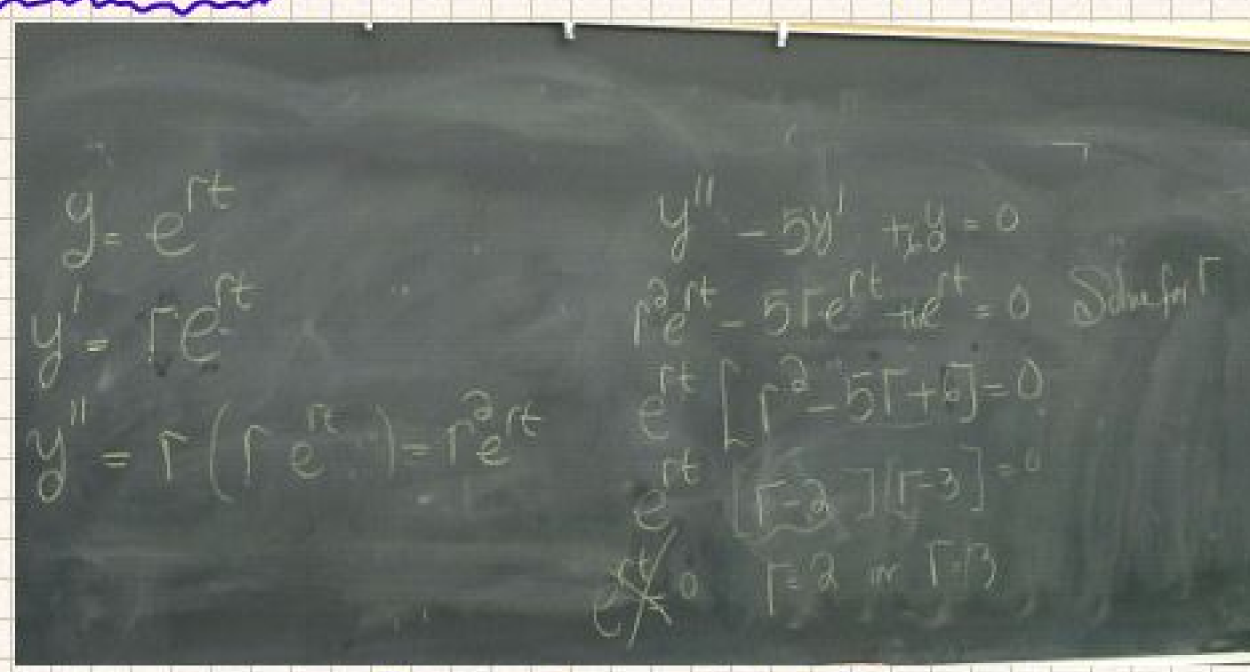
$x' = \frac{1}{2} \quad y' = 2t$

$\frac{2t}{\frac{1}{2}} = 2t(2) = 4t$

$y^2 + x^2 = 1$

$y^2 = 1 - x^2$

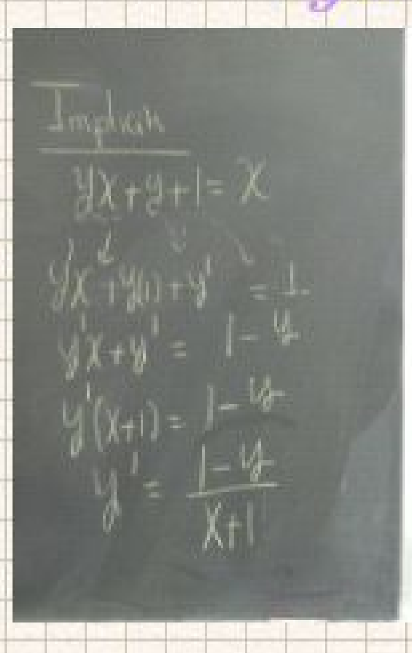
$y = \pm\sqrt{1-x^2}$

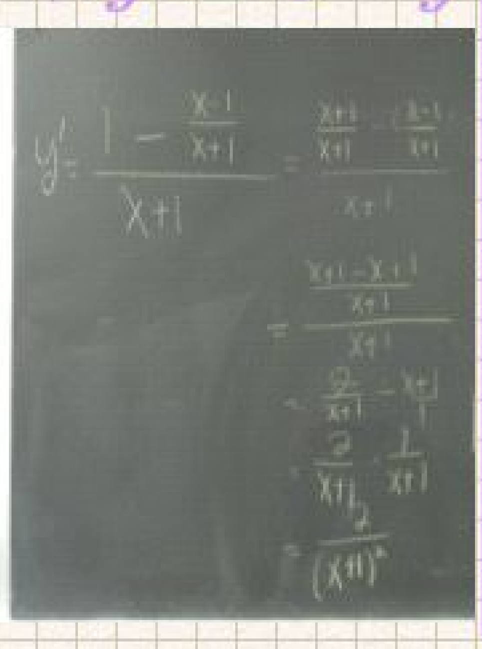


Implicit

$5y^2 + \sin y = x^2$

Implicit	Explicit
$yx + y + 1 = x$	$yx + y + 1 = x$
$y'x + y(1) + y' = 1$	$yx + y = x - 1$
$y'x + y' = 1 - y$	$y(x+1) = x - 1$
	$y = \frac{x-1}{x+1}$





$$\frac{d}{dx}[x^n] = nx^{n-1}$$

$$\frac{d}{dx}[u^n] = nu^{n-1}\frac{du}{dx}$$

$u = f(x)$

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + g'(x)f(x)$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)[f'(x)] - [f(x)g'(x)]}{[g(x)]^2}$$

$$\frac{d}{dx}[f^{-1}(x)] = \frac{1}{f'[f^{-1}(x)]}$$

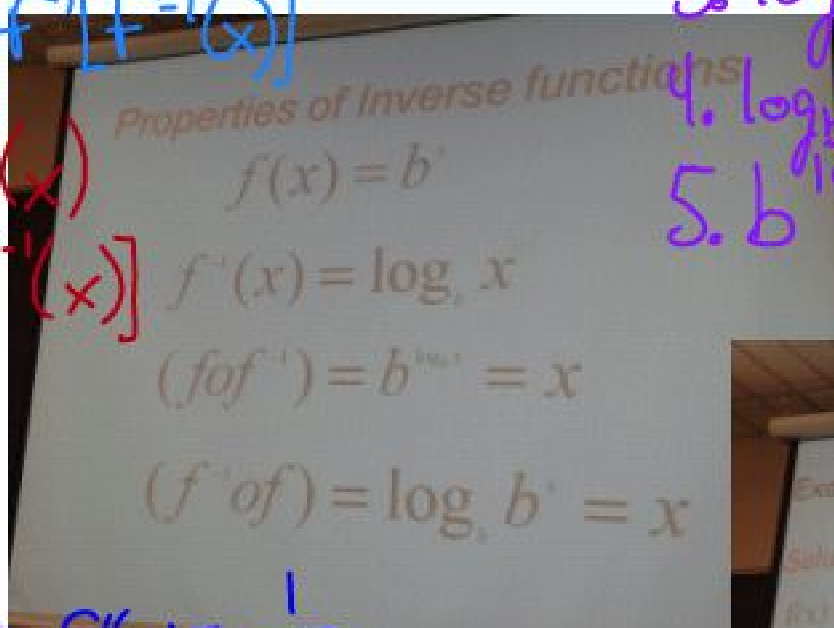
$F(y) = y\ln[1+e^y]$

$F'(y) = 1\cdot\ln[1+e^y] + \frac{y\cdot e^y}{1+e^y}$

$a^y = x \Leftrightarrow \log_a x = y$

1. $\log_b x = y$ iff $b^y = x$
2. $\log_b 1 = 0$
3. $\log_b b = 1$
4. $\log_b b^x = x$
5. $b^{\log_b x} = x$

9. $\log_b M^p = p\log_b M$



let $y = f^{-1}(x)$

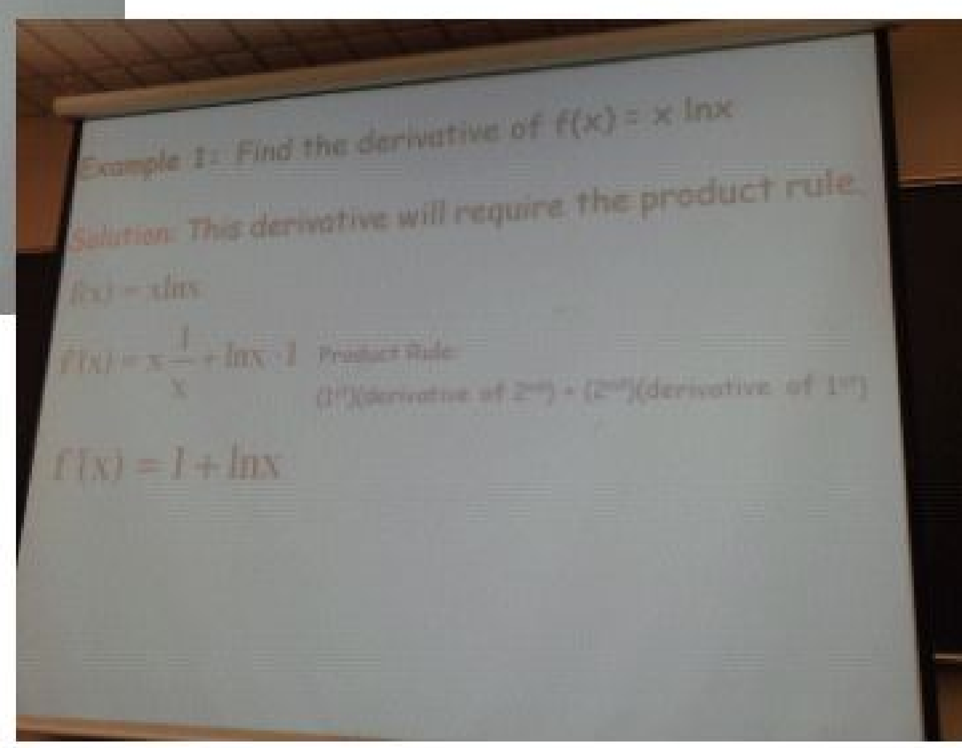
$f(y) = f[f^{-1}(x)]$

$f(y) = x$

$f'(y) = 1$

$f(x) = \ln x \rightarrow f'(x) = \frac{1}{x}$

$f(x) = \log_b x \rightarrow f'(x) = \frac{1}{x\ln b}$



#2 $f(x) = x\ln x - x$

$\left[x\left(\frac{1}{x}\right) + \ln x\right] - 1$

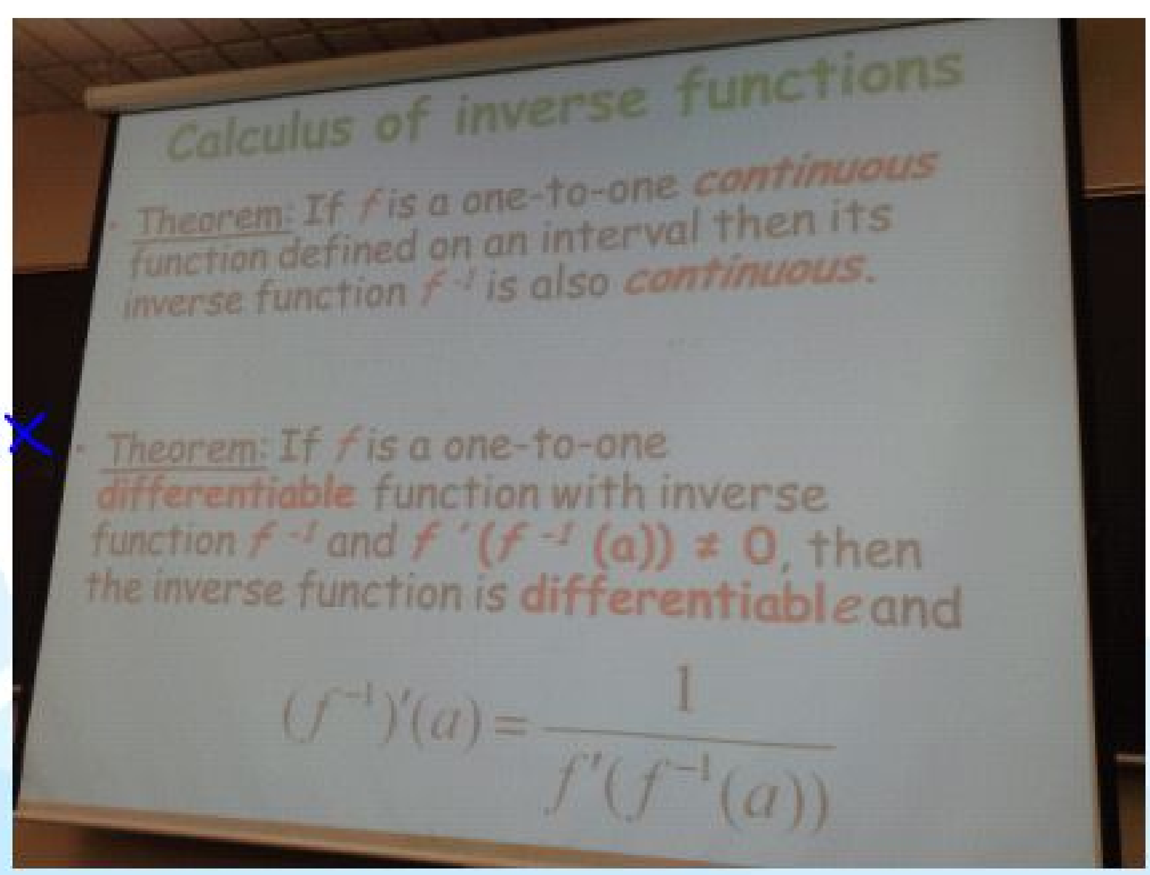
$\frac{x}{x} + \ln x - 1 = \boxed{\ln x}$

$f(x) = \ln[\sin^2 x]$

$= \ln[(\sin x)^2]$

$\frac{d}{dx} \rightarrow \frac{1}{[\sin^2 x]} \cdot 2\sin x \cdot \cos x$

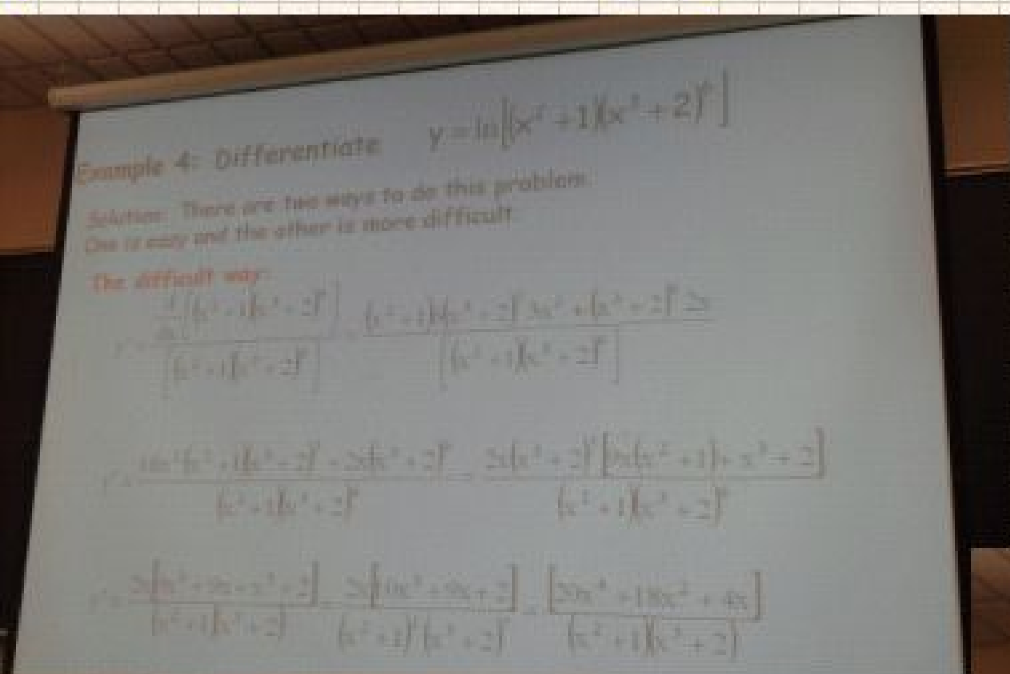
$\frac{2\sin x\cos x}{[\sin x]^2} \rightarrow$

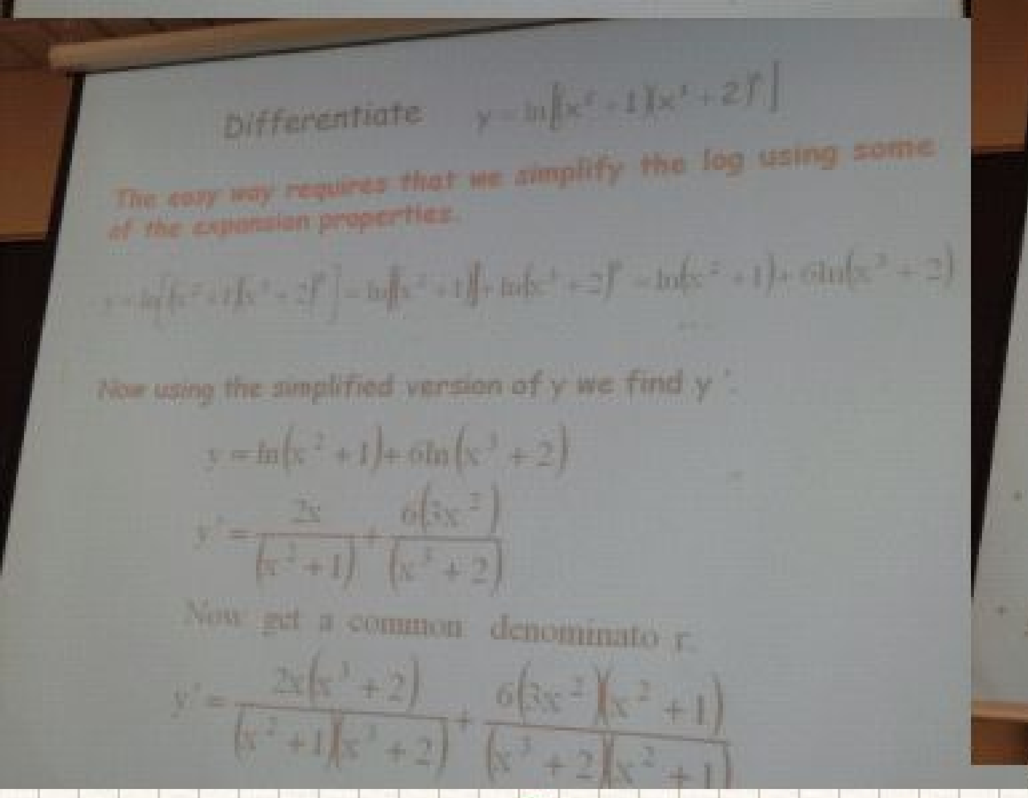


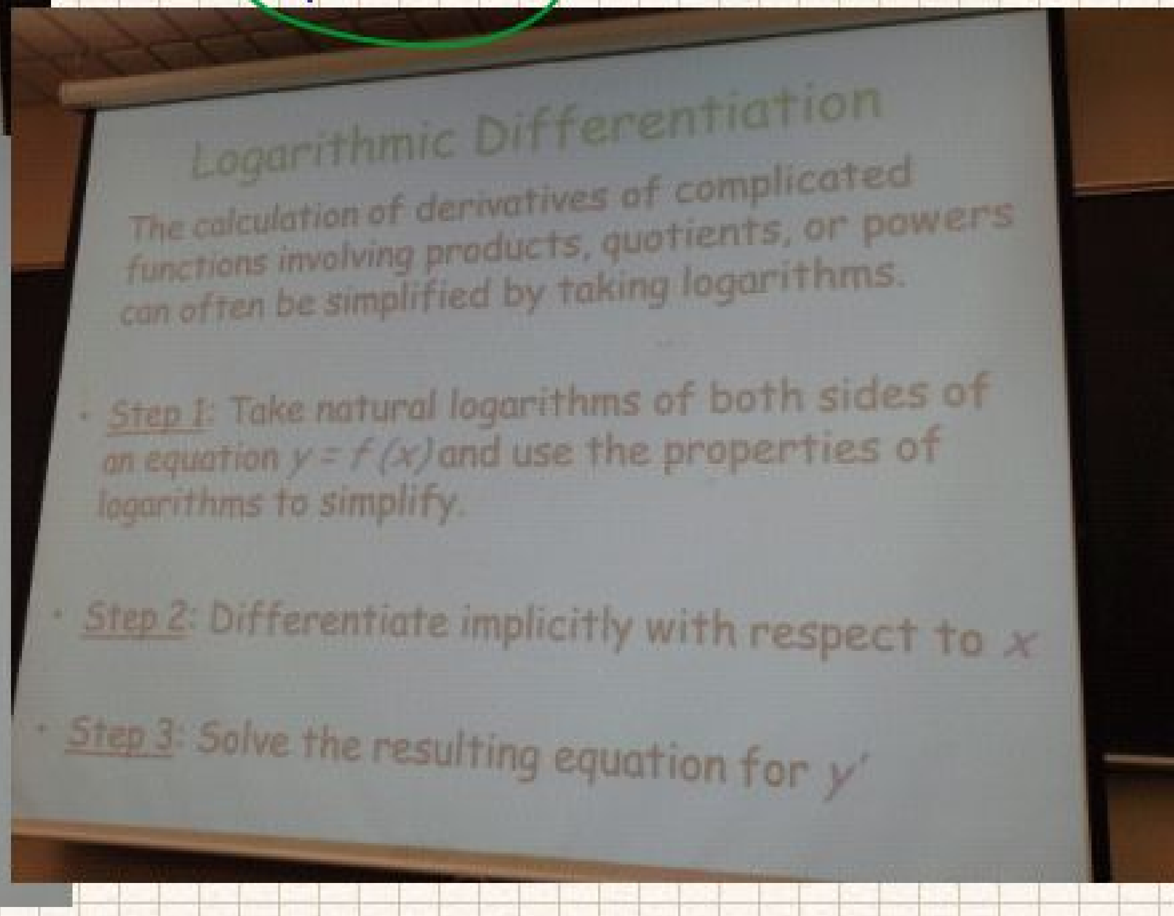
$$g(x) = \frac{\ln x}{x}$$

$$y = \ln[(x^2+1)(x^3+2)^6]$$

$$g(t) = \ln[t^2 \cdot e^{-t^2}]$$
$$= \ln[t^2] + \ln[e^{-t^2}]$$
$$= 2\ln[t] - t^2 \ln e$$
$$g(t) = 2\ln t - t^2$$
$$g'(t) = 2 \cdot \frac{1}{t} - 2t$$
$$g'(t) = \frac{2}{t} - 2t$$

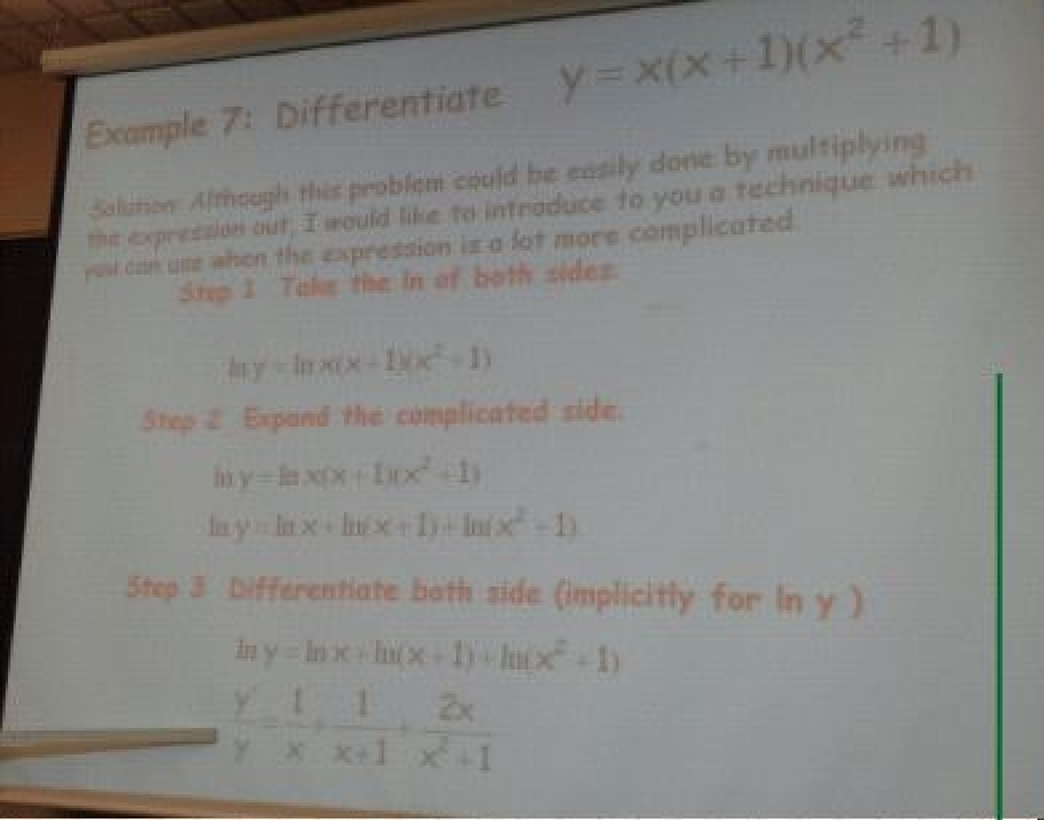
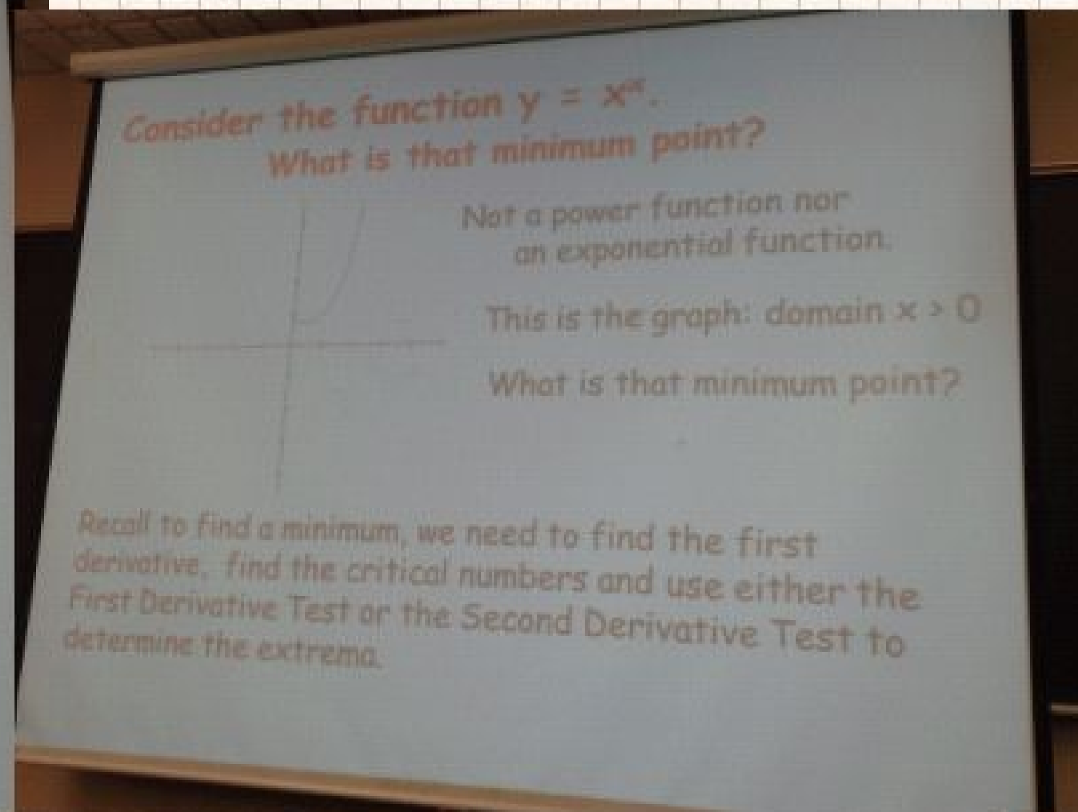
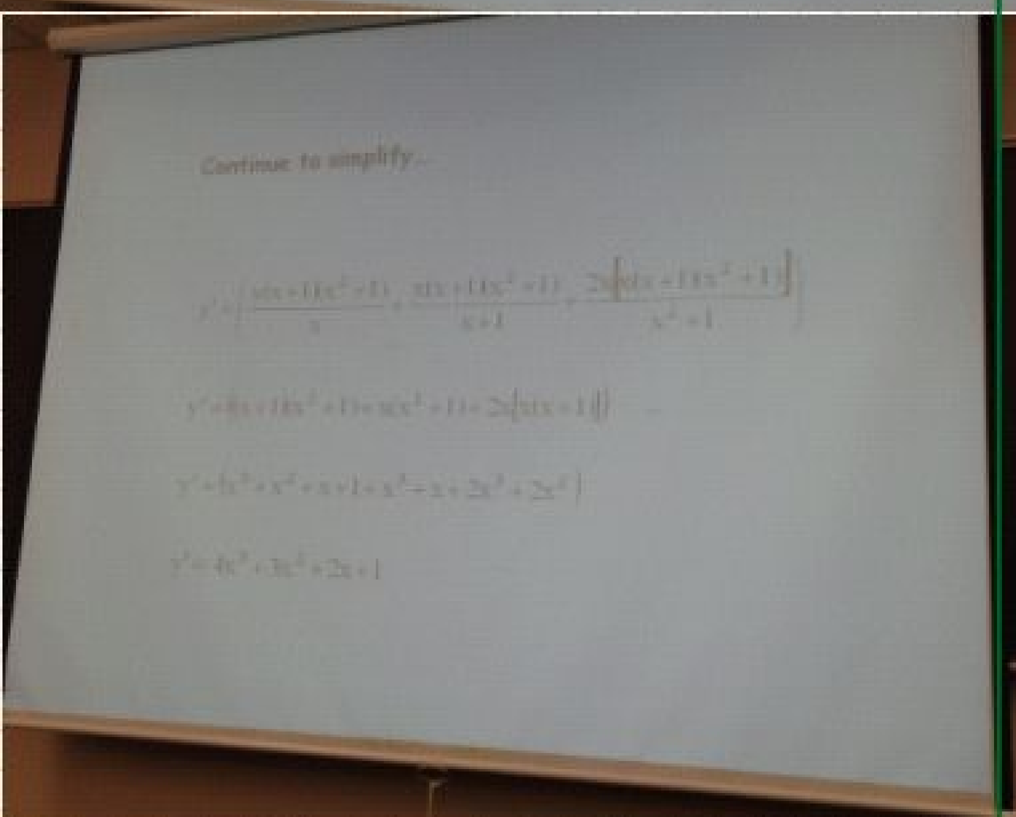
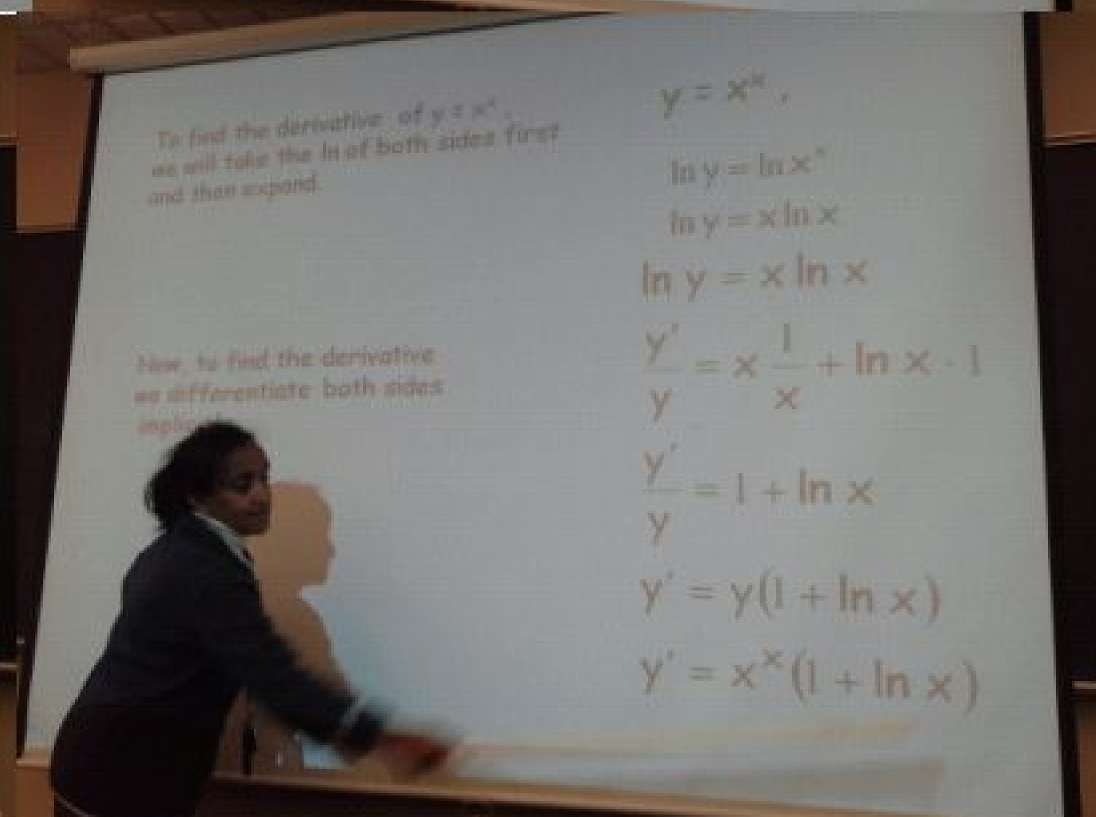
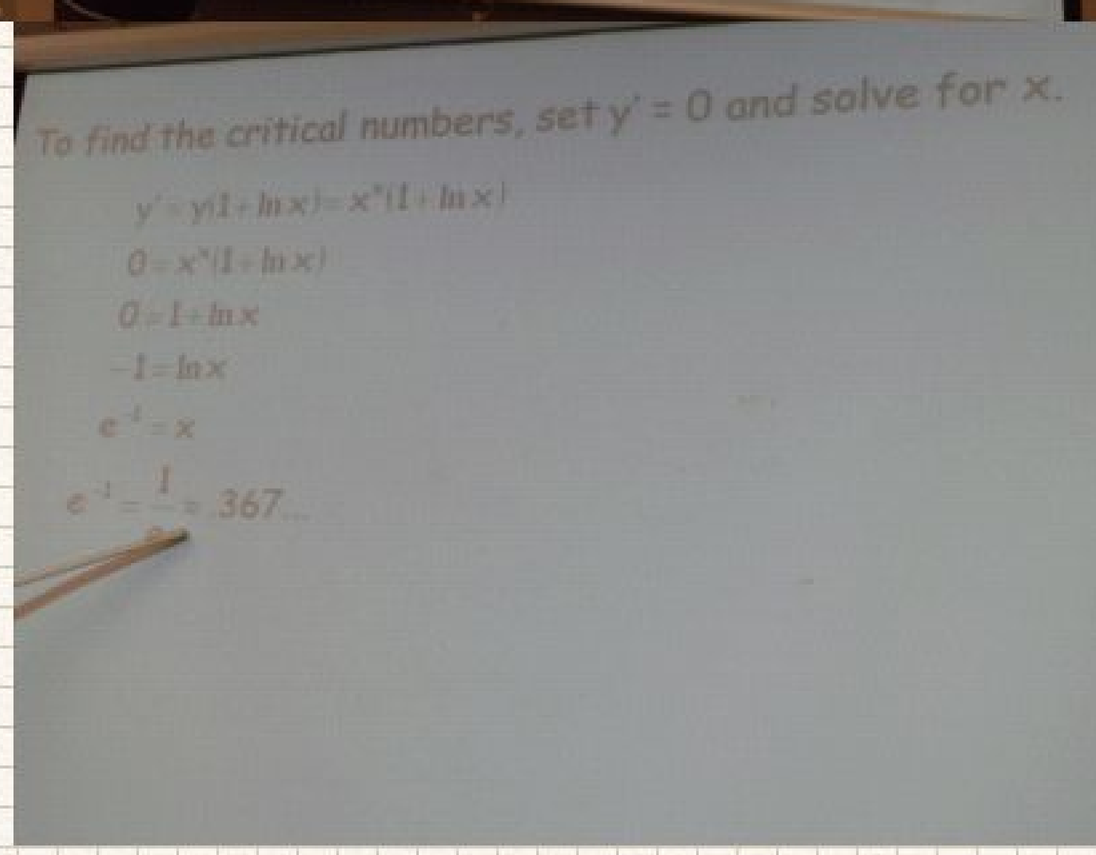


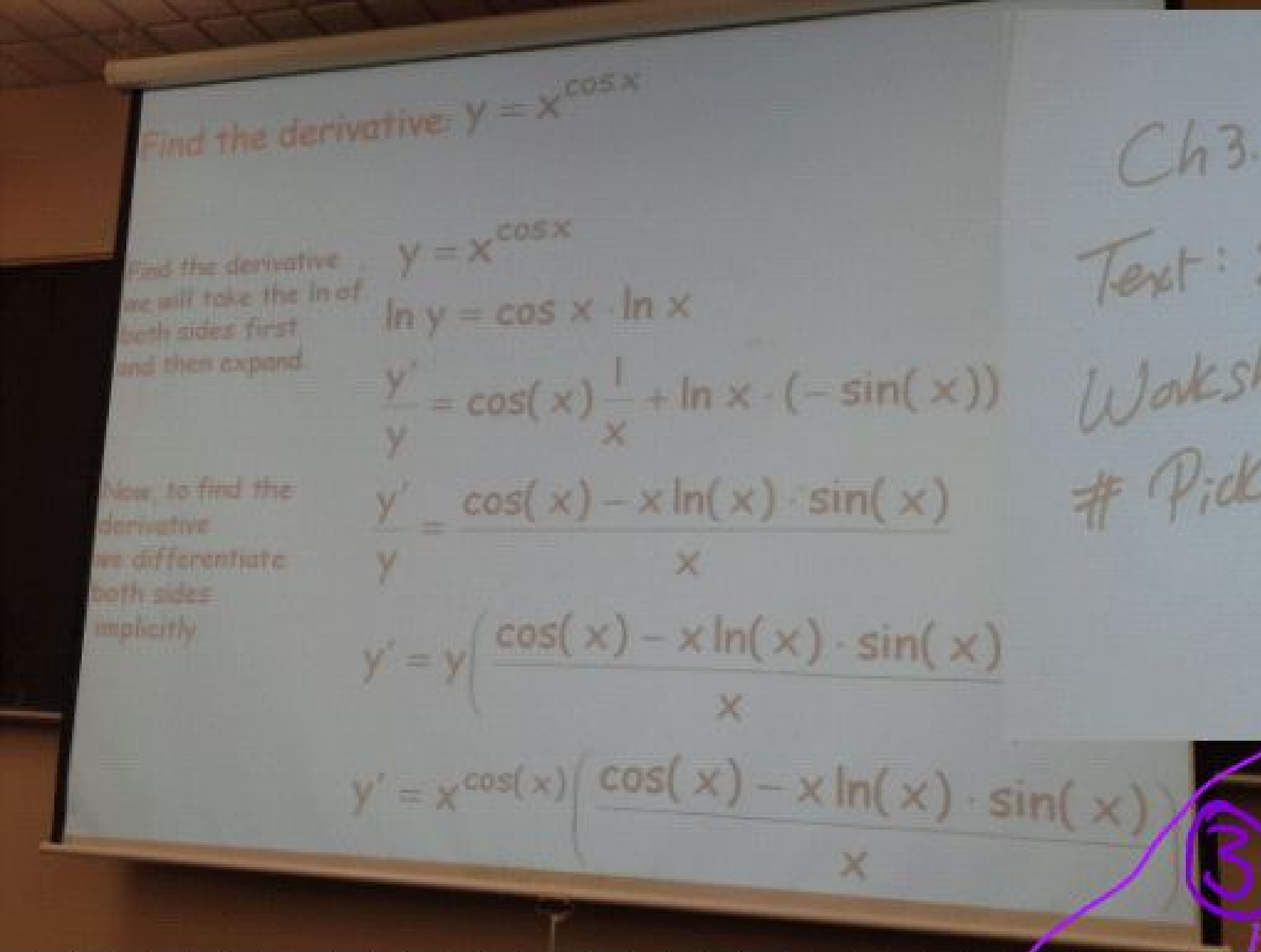


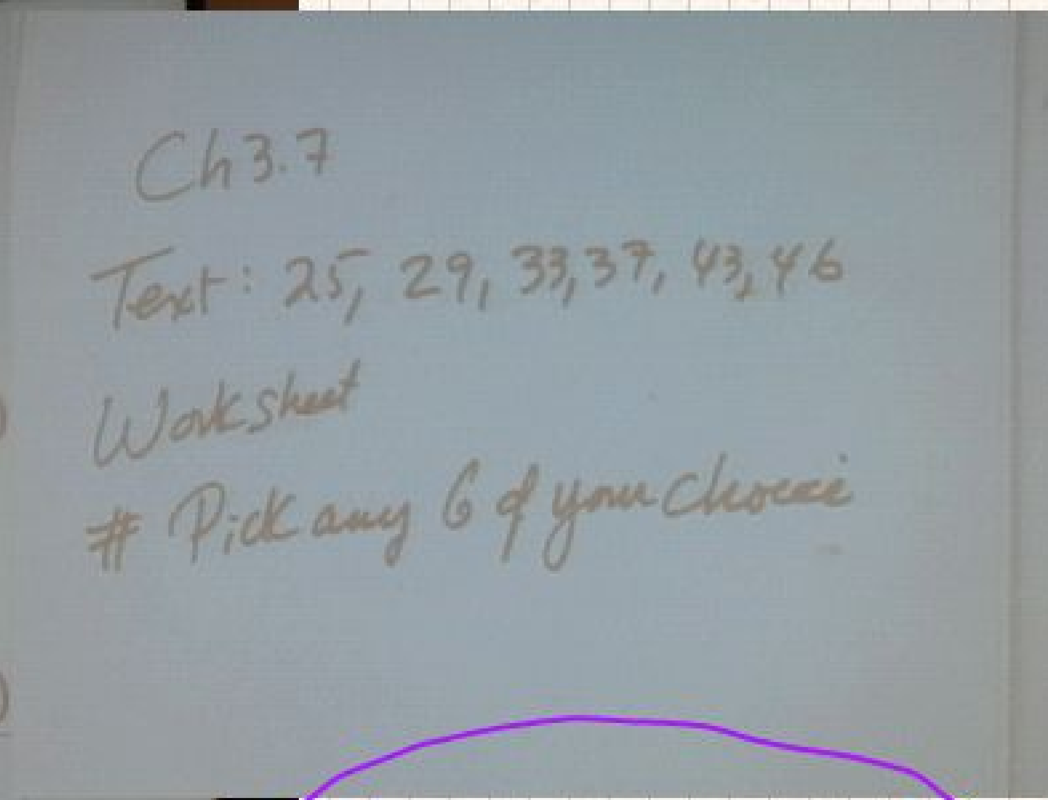


$$f(x) = (2x-1)^2$$
$$= 4x^2 - 4x + 1$$
$$\frac{df}{dx} = 8x - 4$$

$$f(x) = (2x-1)^2$$
$$y = (2x-1)^2$$
$$\ln y = \ln(2x-1)^2$$
$$\ln y = 2\ln(2x-1)$$
$$\frac{1}{y} y' = 2 \cdot \frac{1}{2x-1} \cdot 2$$


Example 7: Differentiate $y = x(x+1)(x^2+1)$
Solution: Although this problem could be easily done by multiplying the expression out, I would like to introduce to you a technique which you can use when the expression is a lot more complicated.
Step 1 Take the ln of both sides.
$\ln y = \ln x(x+1)(x^2+1)$
Step 2 Expand the complicated side.
$\ln y = \ln x(x+1)(x^2+1)$
$\ln y = \ln x + \ln(x+1) + \ln(x^2+1)$
Step 3 Differentiate both side (implicitly for ln y)
$\ln y = \ln x + \ln(x+1) + \ln(x^2+1)$
$\frac{y'}{y} = \frac{1}{x} + \frac{1}{x+1} + \frac{2x}{x^2+1}$

Consider the function $y = x^x$.
What is that minimum point?
Not a power function nor an exponential function.
This is the graph: domain $x > 0$
What is that minimum point?
Recall to find a minimum, we need to find the first derivative, find the critical numbers and use either the First Derivative Test or the Second Derivative Test to determine the extrema.

Continue to simplify.
$y' = 4x^3 + 3x^2 + 2x + 1$

To find the derivative of $y = x^x$, we will take the ln of both sides first and then expand.
$y = x^x$,
$\ln y = \ln x^x$
$\ln y = x \ln x$
$\ln y = x \ln x$
Now, to find the derivative we differentiate both sides
$\frac{y'}{y} = x\frac{1}{x} + \ln x \cdot 1$
$\frac{y'}{y} = 1 + \ln x$
$y' = y(1 + \ln x)$
$y' = x^x(1 + \ln x)$

To find the critical numbers, set $y' = 0$ and solve for x.
$y' = y(1+\ln x) = x^x(1+\ln x)$
$0 = x^x(1+\ln x)$
$0 = 1+\ln x$
$-1 = \ln x$
$e^{-1} = x$
$e^{-1} = \frac{1}{e} = .367...$

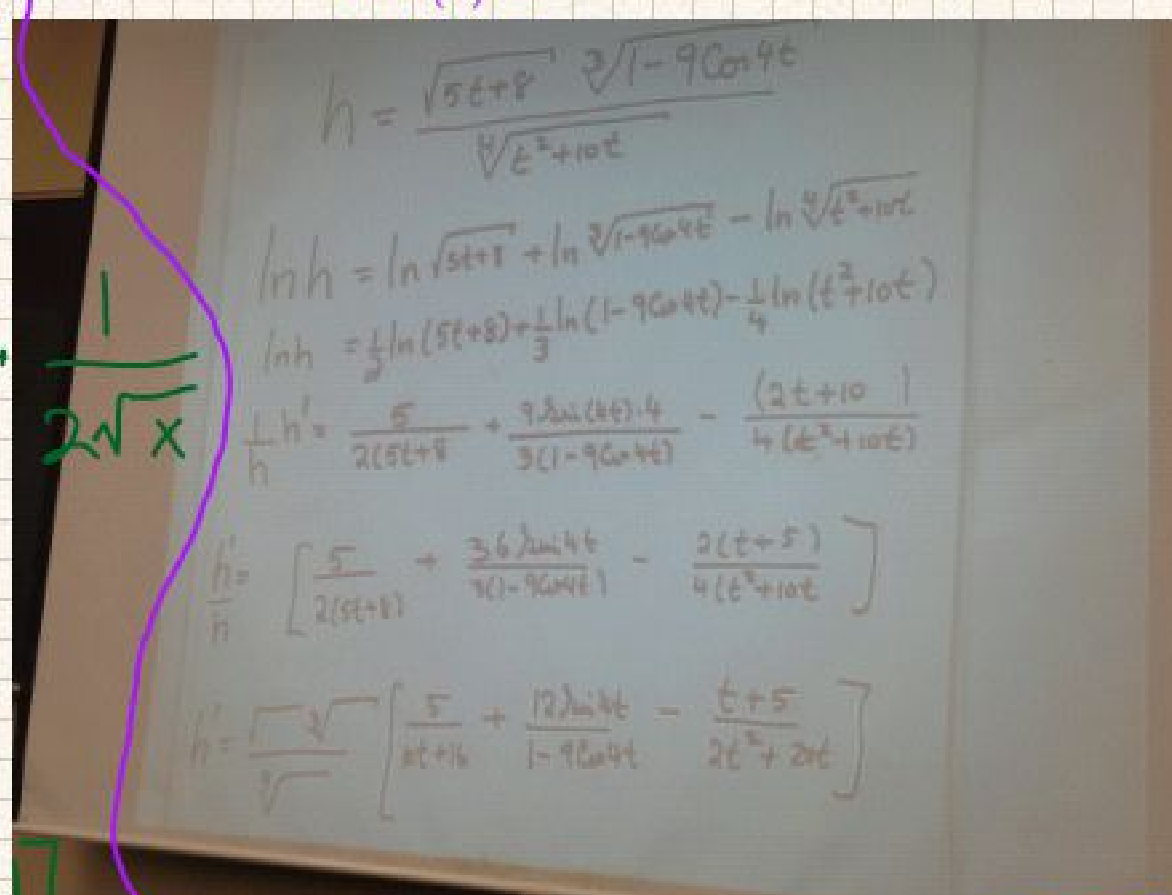




3

$$h(t) = \frac{\sqrt{5t+8}\,\sqrt[3]{1-9\cos(4t)}}{\sqrt[4]{t^2+10t}}$$

$$\ln h(t) =$$



$$y = \sqrt{x}^x$$

- $\ln y = \ln[\sqrt{x}]^x$
- $\ln y = x\ln[\sqrt{x}]$

$$\frac{y'}{y} = 1\cdot\ln[\sqrt{x}] + x\cdot\frac{1}{\sqrt{x}}\cdot\frac{1}{2\sqrt{x}}$$

$$\frac{y'}{y} = \ln\sqrt{x} + \frac{x}{2x}$$

$$y' = y\left[\ln\sqrt{x} + \tfrac{1}{2}\right]$$

$$y' = \sqrt{x}^x\cdot\left[\frac{2\ln\sqrt{x}+1}{2}\right]$$

$$Q(v) = \frac{2}{\left(6+2v-v^2\right)^4}$$
$$= 2\left(6+2v-v^2\right)^{-4}$$

(11) Find $y'' = \dfrac{d^2y}{dx^2} = ?$

given $6y - xy^2 = 1$

So far, we know

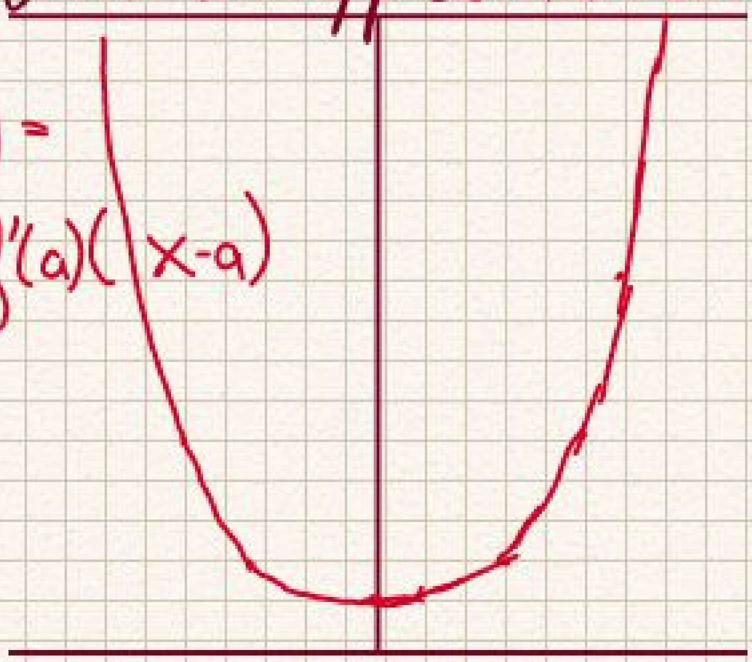
3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7

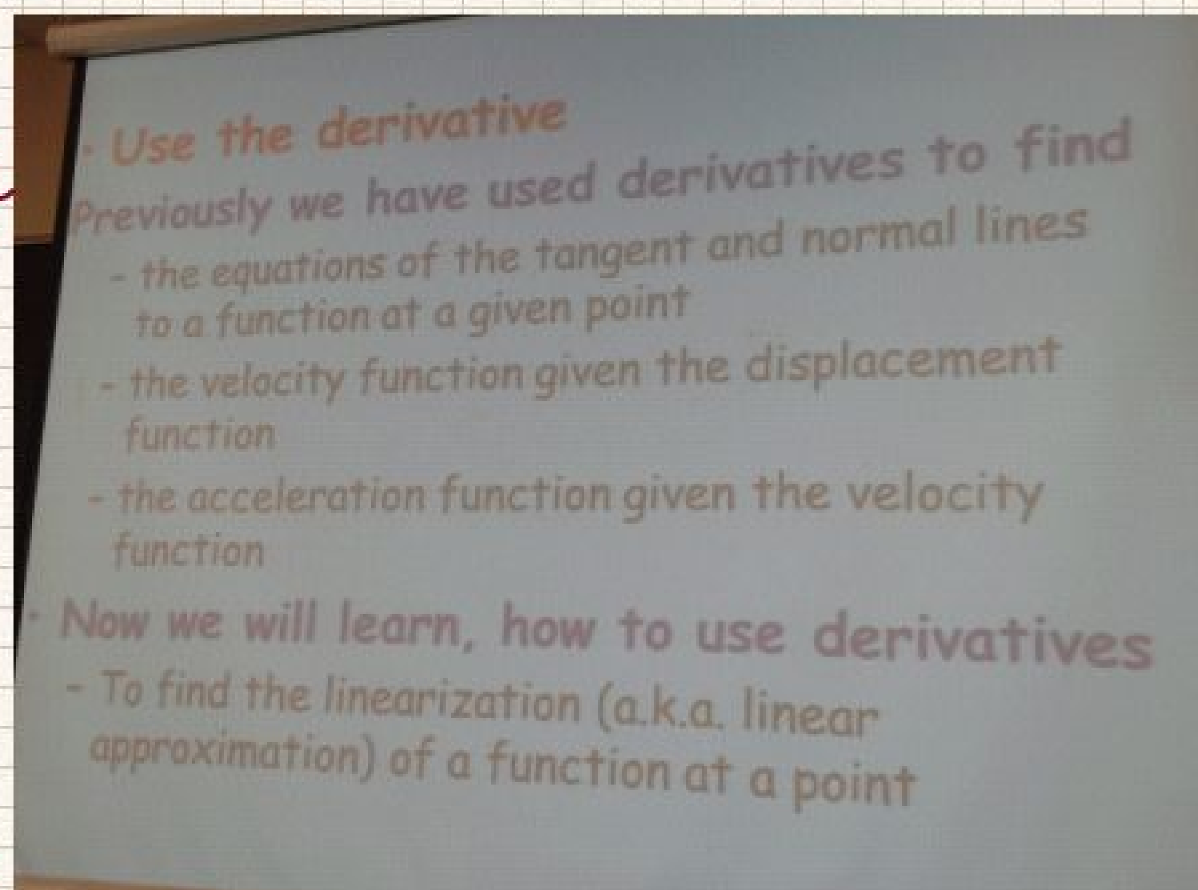
Now we skip 3.8 & begin

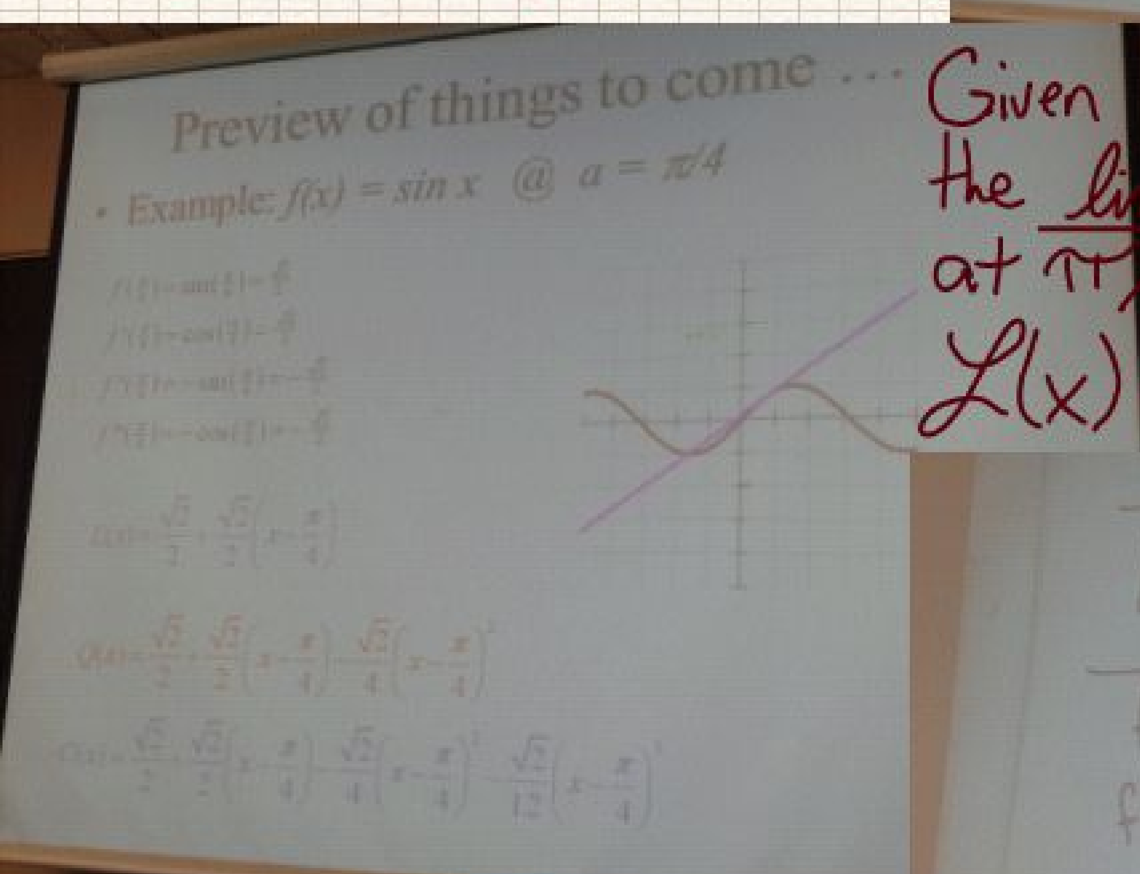
3.9 Linear Approximation

$L(x) =$

$f(a) + f'(a)(x-a)$

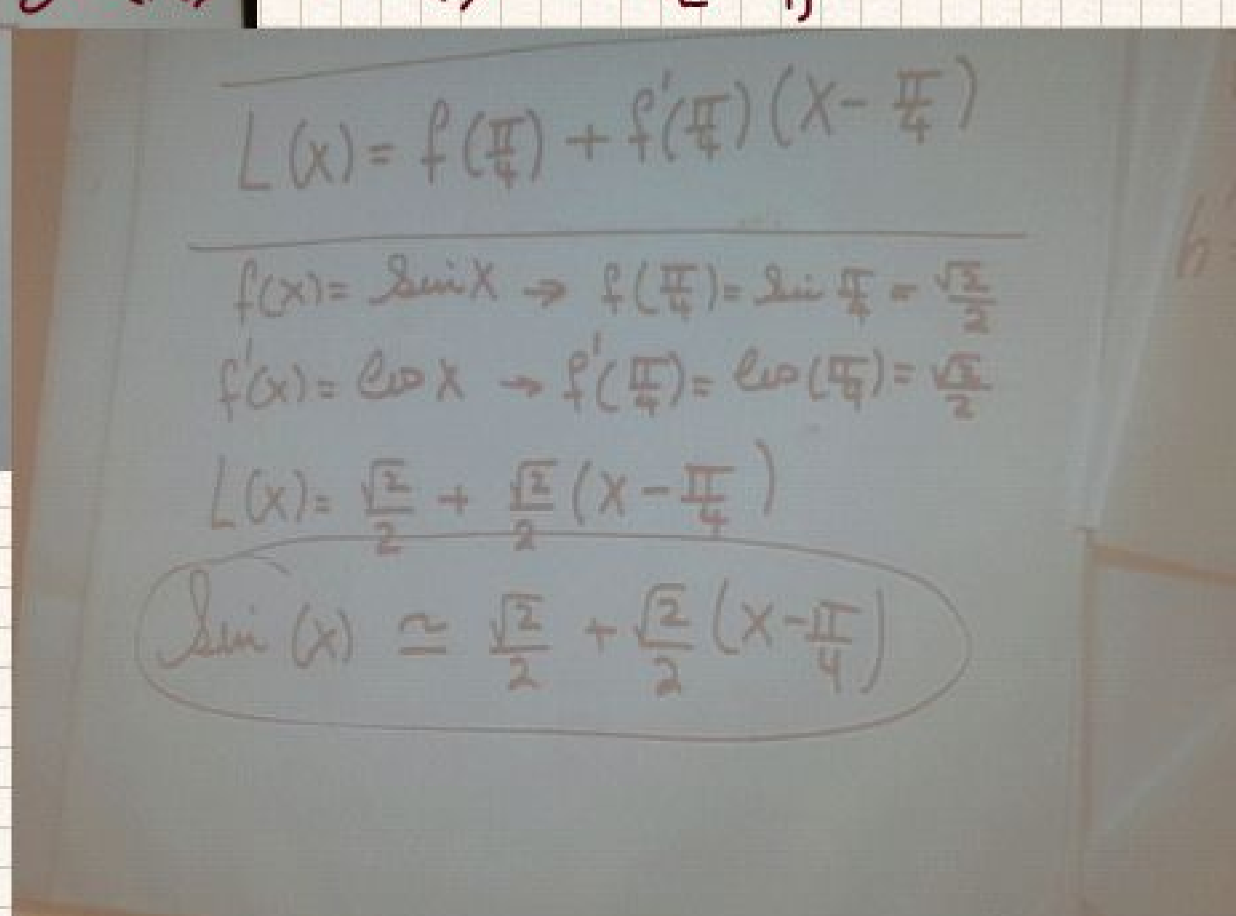


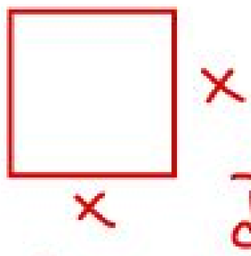




Given $f(x) = \sin(x)$, find the linear approximation at $\pi/4$

$\mathcal{L}(x) = f\left(\frac{\pi}{4}\right) = f'\left(\frac{\pi}{4}\right)\left[x - \frac{\pi}{4}\right]$





□ 3.9 syllabus.
□ Takehome Quiz

$x = 10\text{ cm}$

$\frac{d}{dx} = \pm \frac{1}{5}\text{ cm}$

Find the estimated error in area

$A = x^2 \quad A(x) = x^2 \quad \frac{dA}{dx} = 2x$

$dA = 2\cdot x\cdot dx$

$2(10)(\pm 1/5) \to dA = \pm 4\text{ cm}^2$

Actual Change $A(10 + \frac{1}{5}) =$

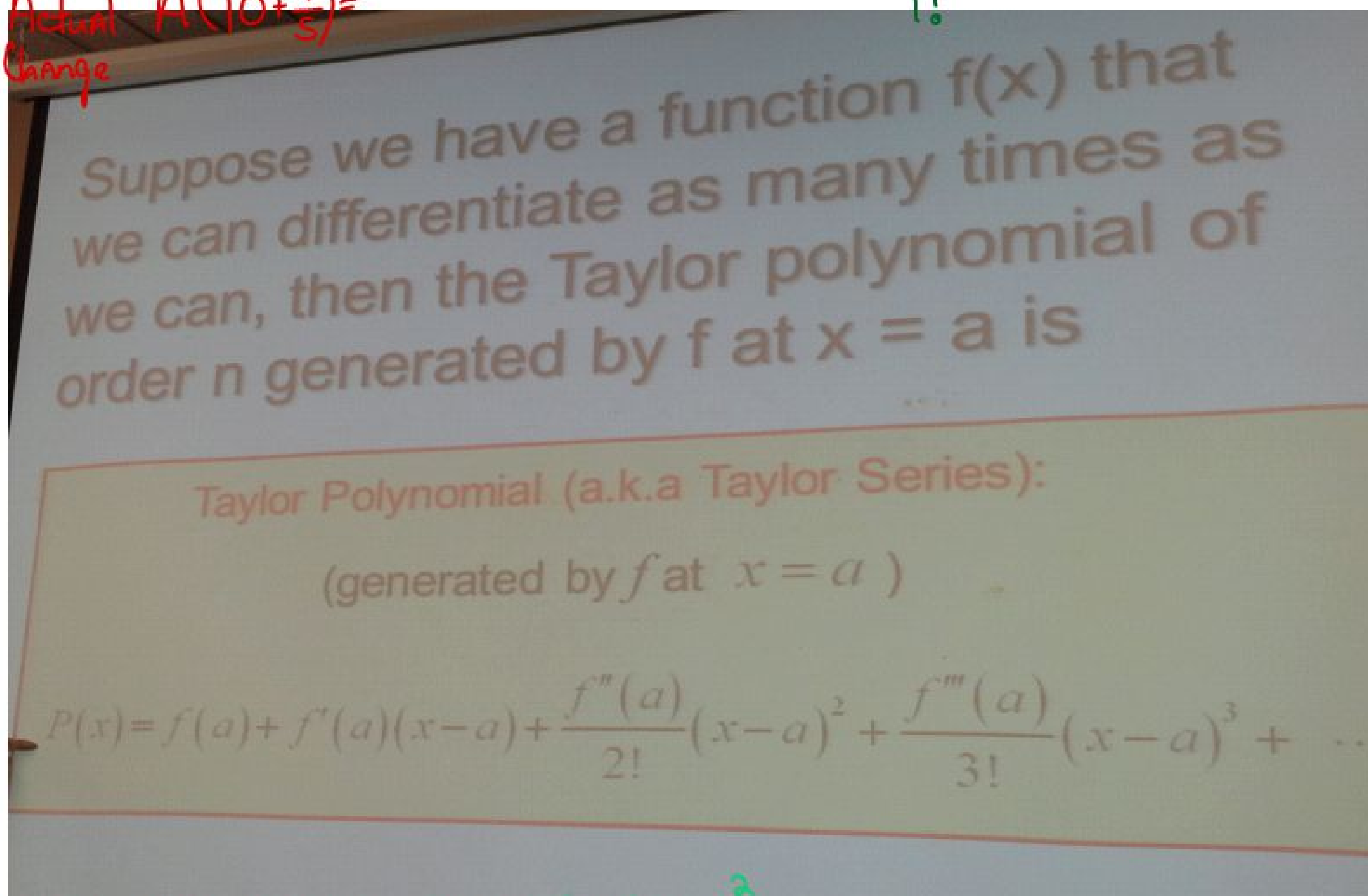
Taylor Polynomials

$f(x) = \sin(x)$, center at 0.

$P_0(x) = f(0) = 0$

$P_0(x) = 0 \quad T_0(x) = 0$

$P_1(x) = f(0) + \frac{f'(0)(x-0)}{1!}$



$P_2(x) = \underbrace{f(0) + f'(0)(x-0)}_{P_1(x)} + \frac{f''(0)(x-0)^2}{2!} = x$

for $f(x) = \sin x$
center at $x = 0$

$P_3(x) = f(0) + f'(0)(x-0) + \frac{f''(0)(x-0)^2}{2!} + \frac{f'''(0)(x-0)^3}{3!}$

Find $\frac{dy}{dx}$?

$y=\sin(\cos(x^2-3x))$

$\frac{dy}{dx}=\cos(\cos(x^2-3x))(-\sin(x^2-3x))(2x-3)$

$\frac{dy}{dx}$ $y=e^{\sin x} \rightarrow e^{\sin x}(\cos x)$

$\frac{dy}{dx}$ $y=\frac{5^{x^2-1}}{x^5-1} \rightarrow \boxed{\frac{[5^{x^2-1}\cdot \ln 5(2x)\cdot x^5-1]-[(5x^4\cdot 5^{x^2-1})]}{(x^5-1)^2}}$

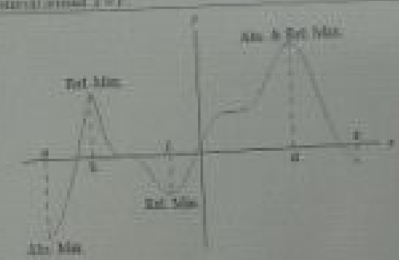
$\frac{dy}{dx}$ $y=\frac{\sqrt[5]{x^2-1}(7x^2-3x+1)^5}{x^4(x+1)}$

Ch 4.2

Definition

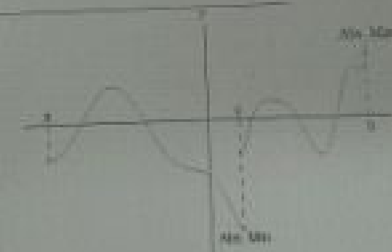
1. We say that $f(x)$ has an **absolute (or global) maximum** at $x=c$ if $f(x)\le f(c)$ for every x in the domain we are working on.
2. We say that $f(x)$ has a **relative (or local) maximum** at $x=c$ if $f(x)\le f(c)$ for every x in some open interval around $x=c$.
3. We say that $f(x)$ has an **absolute (or global) minimum** at $x=c$ if $f(x)\ge f(c)$ for every x in the domain we are working on.
4. We say that $f(x)$ has a **relative (or local) minimum** at $x=c$ if $f(x)\ge f(c)$ for every x in some open interval around $x=c$.

The maximum & minimum points of function are collectively called the Extrema of the function.



Extreme Value Theorem

Suppose that $f(x)$ is continuous on the interval $[a,b]$ then there are two numbers $a\le c, d\le b$ so that $f(c)$ is an absolute maximum for the function and $f(d)$ is an absolute minimum for the function.



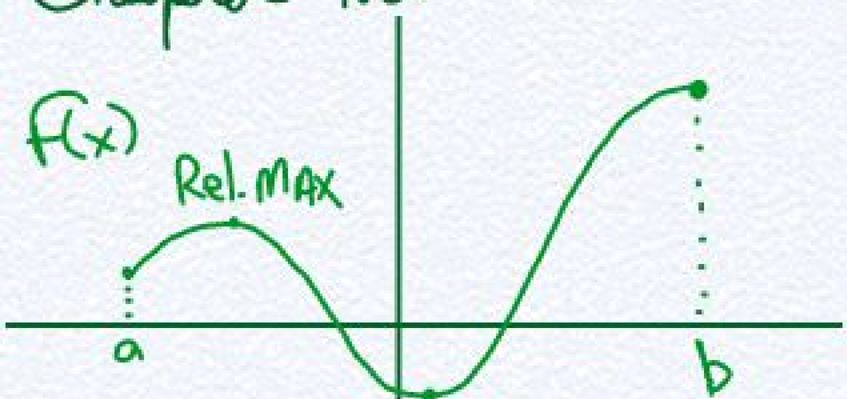
the graph is discontinuous at $x=c$, yet it has an absolute mini at $x=c$

Fermat's Theorem

If $f(x)$ has a relative extrema at $x=c$ and $f'(c)$ exists then $x=c$ is a critical point of $f(x)$. In fact, it will be a critical point such that $f'(c)=0$.

What is a critical point?
How do you find it?
How do you find the Absolute Max & Mini on a closed interval?

Chapter 4.2



$f(x)=6x^5+33x^4-30x^3+100$

find the critical points:

$\frac{d}{dx}f(x)=30x^4+132x^3-90x^2$

$f(x)=x^2(30x^2+132x-90)$

$x^2=0 \quad 30x^2+132x-90=0$

$x=0,\ x=3/5,\ x=-5$

finding Extrema on a closed Interval:

Given $f(x)$ on $[a,b]$

Step: Find $f'(x)$

Step: Find the critical points.

$f'(x)=0$ or $f'(x)=$ undefined.

Step. Evaluate the function at the End points

$x=a$	$f(a)$
$x=b$	$f(b)$

at the critical points

$x=c$	$f(c)$
$x=d$	$f(d)$

$f(x)=\sqrt[5]{x^2-6x}=(x^2-6x)^{\frac{1}{5}}$

$f'(x)=\frac{1(2x-6)}{5(x^2-6x)^{\frac{4}{5}}}$

$f'(x)=\frac{2x-6}{5\cdot\sqrt[5]{(x^2-6x)^4}}$

$f'(x)=0$
$2x-6=0$
$x=3$

$f'(x)=$ undefined
$x^2-6x=0$
$x=0,\ x=6$

HW 19Nov12

1. finish 4.2
2. Read 4.3
3. Project #2 Syllabus Chapter 4.4

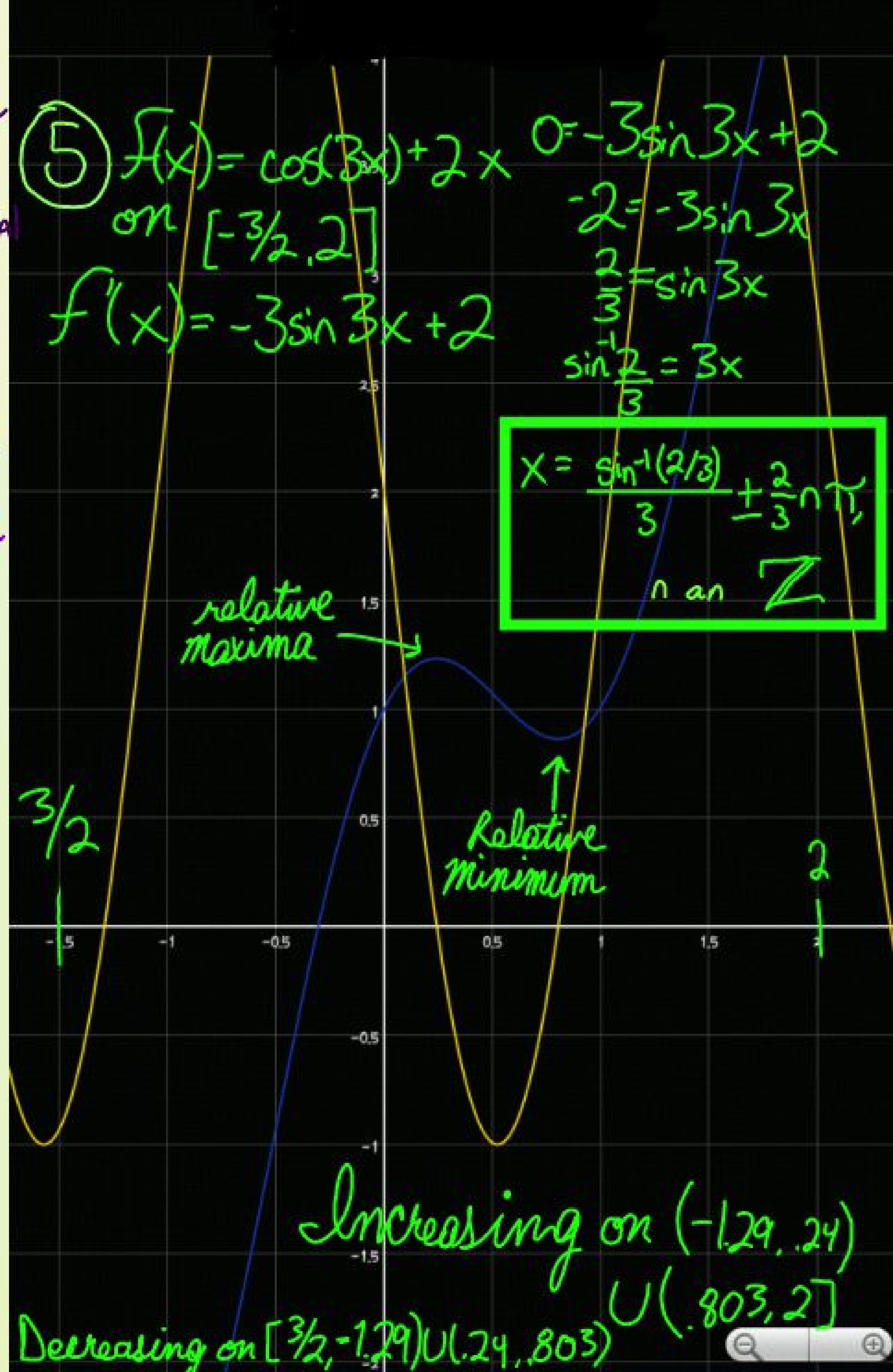
Applications of Derivatives

Chapter 4.2 Extrema on an Interval

- Extreme Value Theorem
- A function may not have Extrema
- critical values
- Fermat's Theorem
- Concave up/down
- Inflection points

$f'(c)=0$
$f''(c)<0$
max at $x=c$

$f'(c)=0$
$f''(c)>0$
relative minima



⑤ $f(x)=\cos(3x)+2x$ on $[-3/2,2]$

$f'(x)=-3\sin 3x+2$

$0=-3\sin 3x+2$

$-2=-3\sin 3x$

$\frac{2}{3}=\sin 3x$

$\sin^{-1}\frac{2}{3}=3x$

$$\boxed{x=\frac{\sin^{-1}(2/3)}{3}\pm\frac{2}{3}n\pi,\quad n \text{ an } \mathbb{Z}}$$

relative maxima

Relative minimum

3/2

2

Increasing on $(-1.29, .24) \cup (.803, 2]$

Decreasing on $[3/2, -1.29) \cup (.24, .803)$

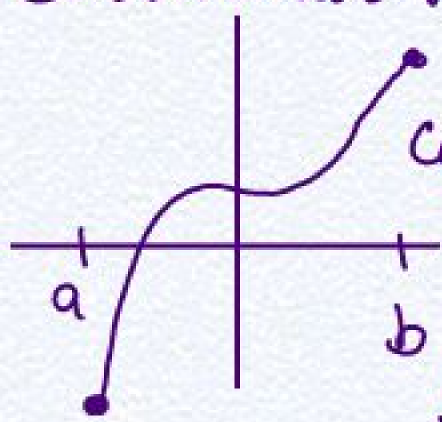
⑫ $f(x)=-x^3+6x^2+12$

$f'(x)=-3x^2+12x$

$f''(x)=-6x+12$

Homework: 4.3 WorkSheet #6, 14, 15, 16-19

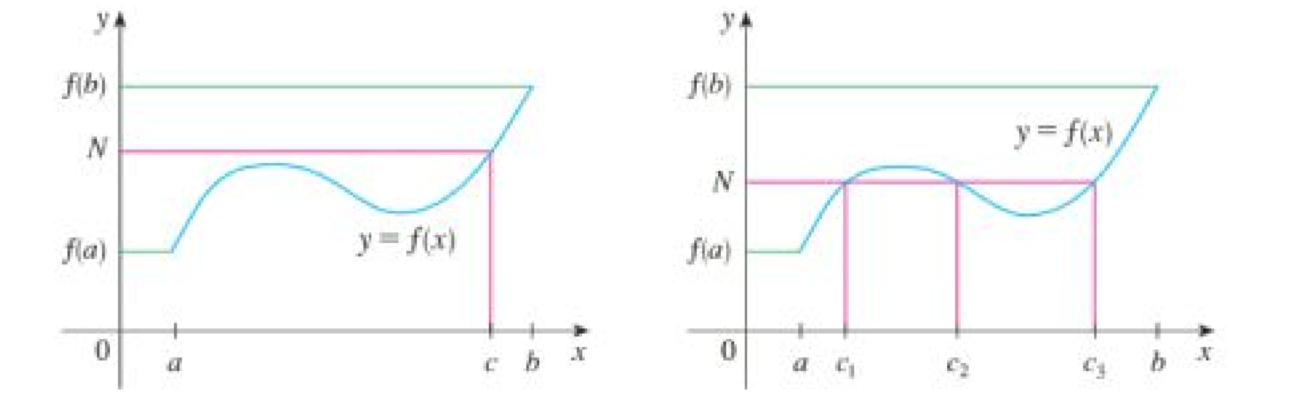
Must Know!
Chapter 2.4 page 120
Intermediate value Theorem



given f(x) continuous on [a,b]
there must be a c at x=0

10 The Intermediate Value Theorem Suppose that f is continuous on the closed interval $[a, b]$ and let N be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then there exists a number c in (a, b) such that $f(c) = N$.

The Intermediate Value Theorem states that a continuous function takes on every intermediate value between the function values $f(a)$ and $f(b)$. It is illustrated by Figure 8. Note that the value N can be taken on once [as in part (a)] or more than once [as in part (b)].

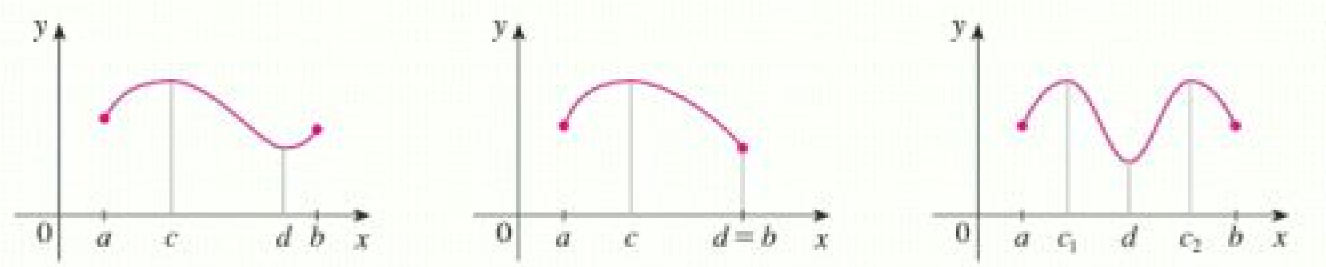


(a) (b)

Chapter 4.2 page 264
Extreme Value Theorem

3 The Extreme Value Theorem If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

The Extreme Value Theorem is illustrated in Figure 7. Note that an extreme value can be taken on more than once. Although the Extreme Value Theorem is intuitively very plausible, it is difficult to prove and so we omit the proof.



Chapter 4.2 Fermat's Theorem:

4 Fermat's Theorem If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.

Chapter 4.3 Mean Value Theorem:

The Mean Value Theorem If f is a differentiable function on the interval $[a, b]$, then there exists a number c between a and b such that

1 $$f'(c) = \frac{f(b) - f(a)}{b - a}$$

or, equivalently,

2 $$f(b) - f(a) = f'(c)(b - a)$$

We can see that this theorem is reasonable by interpreting it geometrically. Figures 1 and 2 show the points $A(a, f(a))$ and $B(b, f(b))$ on the graphs of two differentiable functions.

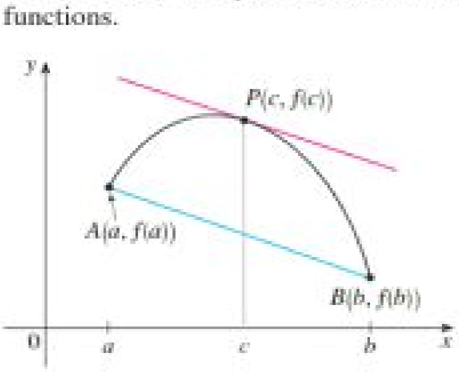


FIGURE 1

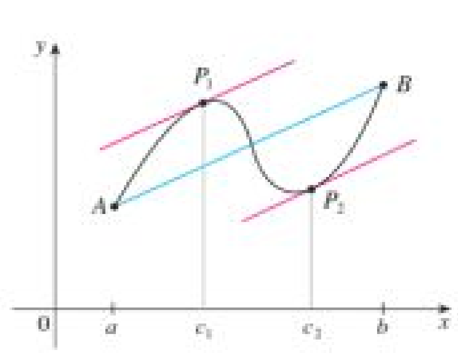


FIGURE 2

The slope of the secant line AB is

$$m_{AB} = \frac{f(b) - f(a)}{b - a}$$

$f(x) = 2x^3 - x^2$ $[0,2]$

find c such that $\frac{f(b)-f(a)}{b-a} = f'(c)$

Step 1: $f(a) = f(0) = 0$

$f(b) = f(2) = 12$

Step 2 = $f'(x) = 6x^2 - 2x$

Step 3 = $f'(c) = 6c^2 - 2c$

$\frac{f(b)-f(a)}{b-a} = 6 \therefore 6 = 6c^2 - 2c$

$0 = 6c^2 - 2c - 6$

Quadratic →

$c = \frac{1 \pm \sqrt{37}}{6}$

$$V = \frac{4}{3}\pi r^3 \quad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

(11) $f(x) = 8x + e^{-3x}$ on $[-2,3]$

$f(-2) = 8(-2) + e^{-3(-2)}$

$-16 + e^6$

$f(3) = 8(3) + e^{-3(3)}$

$24 + e^{-9}$

$\frac{f(b)-f(a)}{b-a} = \frac{24+e^{-9}-(-16+e^6)}{3-(-2)}$

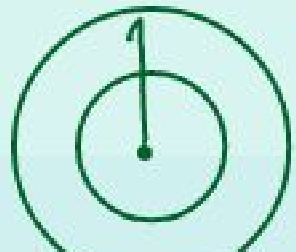
$= \frac{24+e^{-9}+16-e^6}{5}$

$= \frac{40-e^6+e^{-9}}{5} = -72.686$

$\therefore f'(x) = 8 - 3e^{-3x}$

$f(c) = 8 - 3e^{-3c} = -72.686$

~~4.2 syllabus~~
~~4.3 syllabus~~
2.8 1, 10, 15, 19



$\frac{dr}{dt} = .5$ m/sec

$r = 4$m

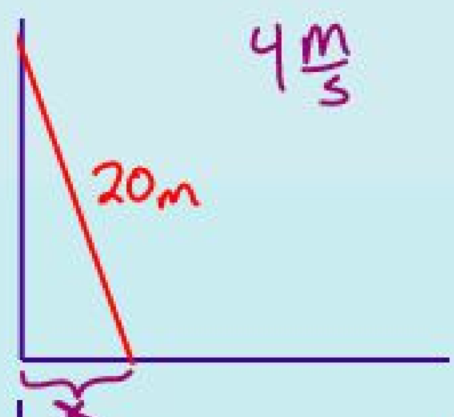
$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$2\pi 4\left(\frac{1}{2}\right)$

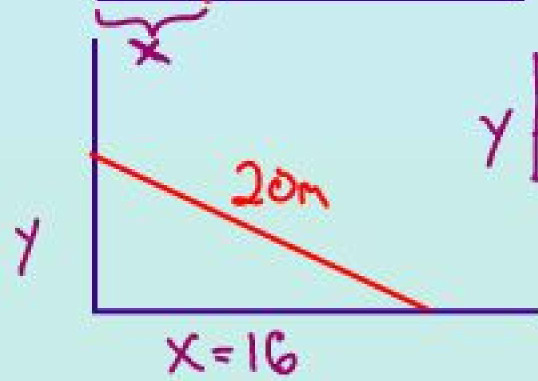
$\frac{dA}{dt} = 4\pi = 12.56$

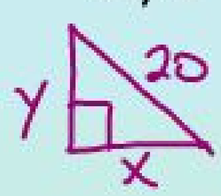
the area of one of these circles is increasing at $4\pi \frac{m^2}{s}$



$4 \frac{m}{s}$

A 20 m ladder leans against a wall and the top slides down at 4 m/s

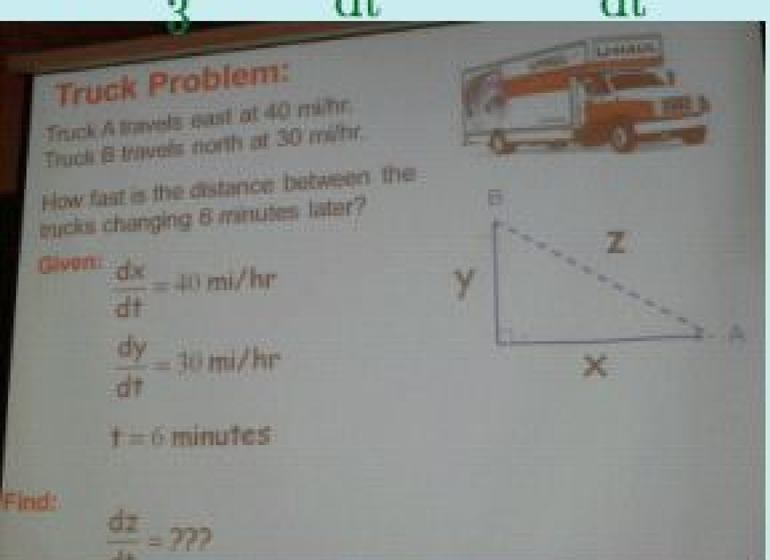


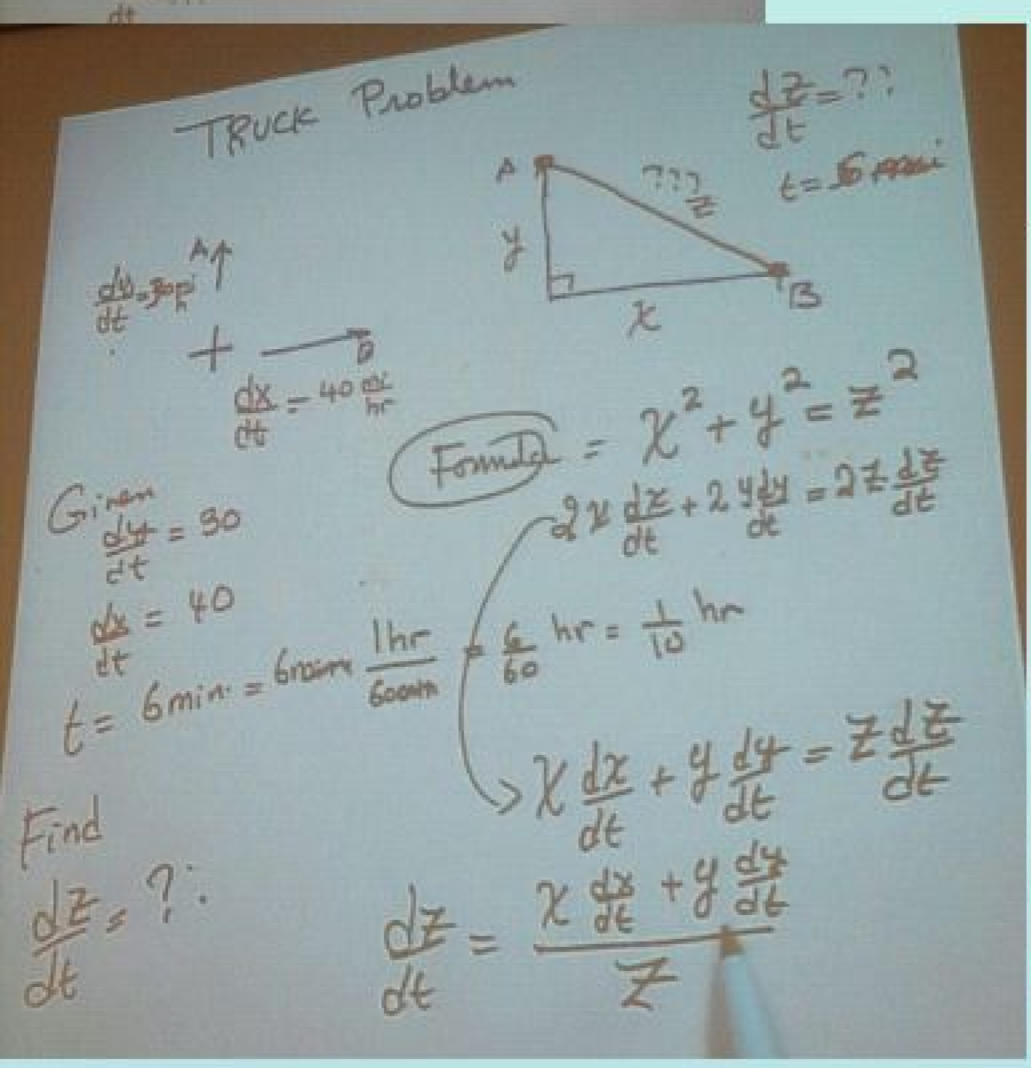


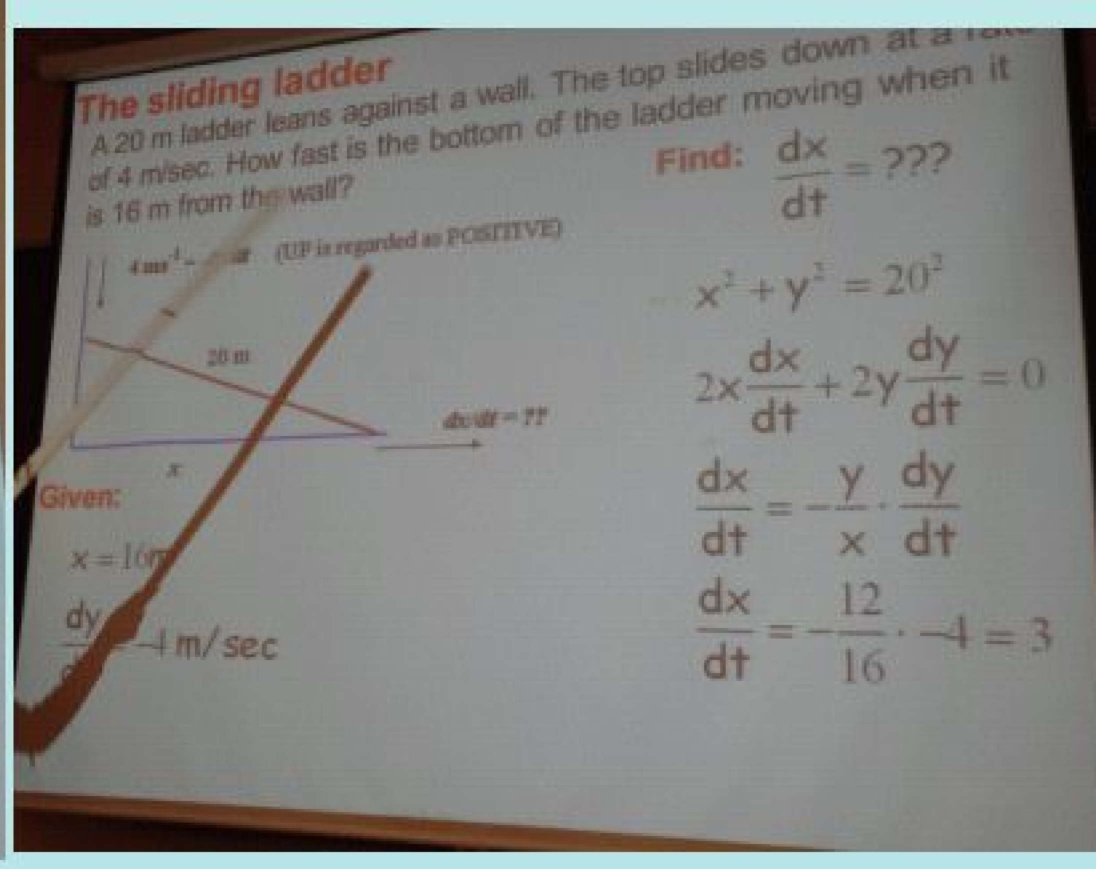
$x^2 + y^2 = 20$

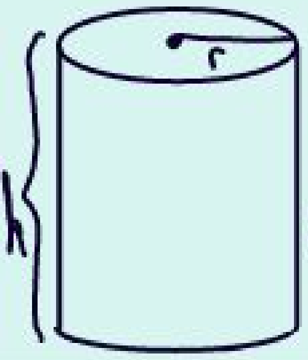
$\frac{dx}{dy}\{x^2 + y^2 = 20\}$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$









Water is draining at 3L/s, How fast is the height changing?

$V = \pi r^2 h$

1 L = 1000 cm^3

$$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$$

$-3L/s = -3000 \frac{cm^3}{s}$

1-10
WS
4.2
4.3

$$-3000 \frac{cm^3}{s} = \pi r^2 \frac{dh}{dt}$$

Given $\theta = \frac{\pi}{4}$ $\frac{d\theta}{dt} = .14 \frac{rad}{min}$

How fast is y changing

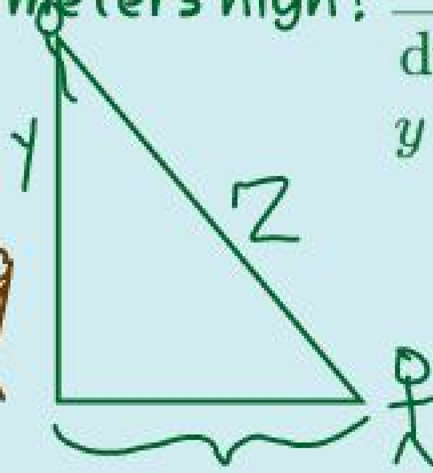
$\tan\theta = \frac{h}{500} = \frac{1}{500} \cdot h$

① find $\frac{dh}{dt}$

A weather balloon is rising vertically at the rate of 5 meters per second. An observer is standing on the ground 300 meters from the point where the balloon was released. At what rate is the distance between the observer and the balloon changing when the balloon is 400 meters high?

$$\frac{dy}{dt} = \frac{+5m}{second}$$

$$y = 400m$$



Formula:

$x^2 + y^2 = z^2$

$300^2 + y^2 = z^2$

$\frac{dy}{dt} \Rightarrow 90{,}000 + y^2 = z^2$

$0 + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

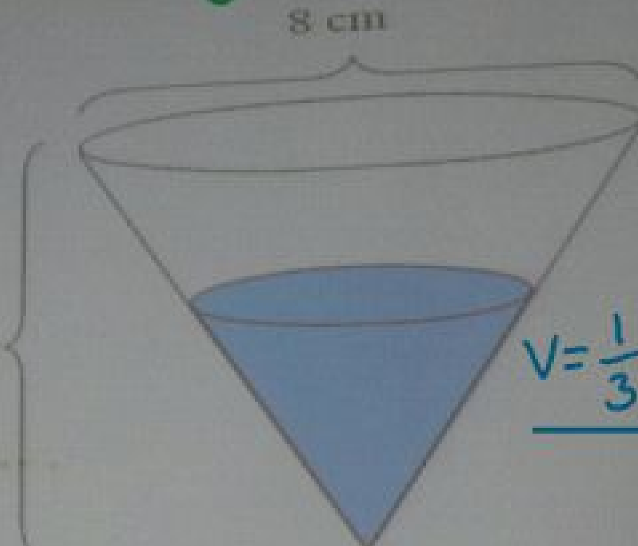
$y\frac{dy}{dt} = z \cdot \frac{dz}{dt}$

$\frac{dz}{dt} = \frac{y}{z}\frac{dy}{dt}$

$\frac{dz}{dt} = \frac{400}{500} \cdot 5 = \boxed{4\frac{m}{s}}$

Draining a tank

A cone filter of diameter 8 cm and height 9 cm is draining at a rate of 2 cm^3/min. Find the rate at which the fluid depth h decreases when $h = 5$ cm.



$V = \frac{1}{3}\pi r^2 h$

cylinder: Volume = area × height
= $\pi r^2 h$

Surface Area = Top + Bottom + side

Similar triangles

$\frac{9}{h} = \frac{4}{r}$

$9r = 4h$

$r = \frac{4}{9}h$

$\frac{dV}{dt} = \frac{-2cm^3}{min}$ find $\frac{dh}{dt}$

$V = \frac{1}{3}\pi r^2 h$

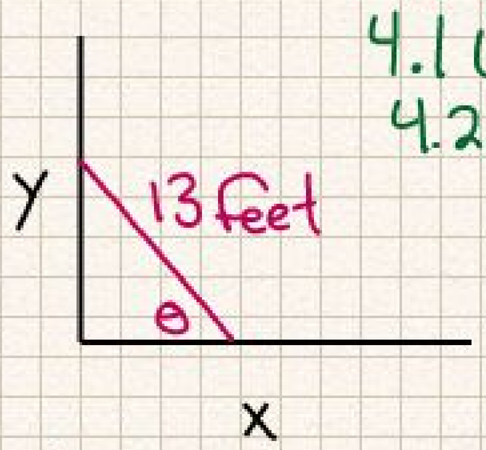
$V = \frac{1}{3}\pi \left(\frac{4}{9} \cdot h\right)^2 \cdot h$

$V = \frac{16}{3 \cdot 81}\pi h^3 \Rightarrow \frac{dV}{dt} = \frac{16}{81}\pi h^2 \frac{dh}{dt} = \frac{-2cm^3}{min}$

$$\frac{-2cm^3}{\frac{16}{81}\pi h^2} = \frac{dh}{dt}$$

4.1 6

Homework:
4.1 Syllabus
4.1 WS
4.2/3 WS



$\frac{dx}{dt} = \frac{3\text{ feet}}{\text{min}}$ find $\frac{d\theta}{dt}$

$x = 12$ feet

$\cos\theta = \frac{x}{13} \Rightarrow -\sin\theta = \frac{1}{13} \cdot \frac{dx}{dt}$

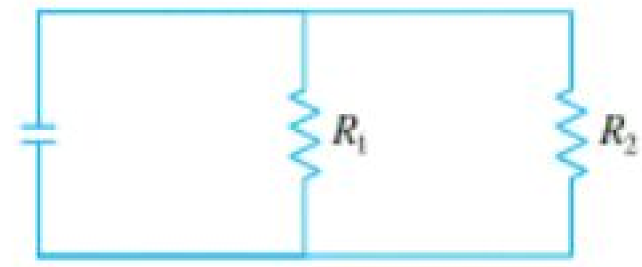
$\frac{d\theta}{dt} = -\frac{1}{13 \cdot \sin\theta} \cdot \frac{dx}{dt}$

$= \frac{-1}{13 \cdot \frac{5}{13}} \cdot 3$

35. If two resistors with resistances R_1 and R_2 are connected in parallel, as in the figure, then the total resistance R, measured in ohms (Ω), is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

If R_1 and R_2 are increasing at rates of 0.3 Ω/s and 0.2 Ω/s, respectively, how fast is R changing when $R_1 = 80\ \Omega$ and $R_2 = 100\ \Omega$?



Given $= R^{-1} = R_1^{-1} + R_2^{-1}$

$\rightarrow -R^{-2} \cdot \frac{dr}{dt} = -R_1^{-2}\left(\frac{dr}{dt}\right) - R_2^{-2}\left(\frac{dr}{dt}\right)$

$\rightarrow \frac{1}{R^2}\frac{dr}{dt} = \frac{1}{R_1^2}\left(\frac{dr}{dt}\right) + \frac{1}{R_2^2}\left(\frac{dr}{dt}\right)$

$\frac{dr}{dt} = R^2\left[\frac{1}{R_1^2}\left(\frac{dr}{dt}\right) + \frac{1}{R_2^2}\left(\frac{dr}{dt}\right)\right]$

4.5 Optimization Problems

2 or more variables
↳ use substitution

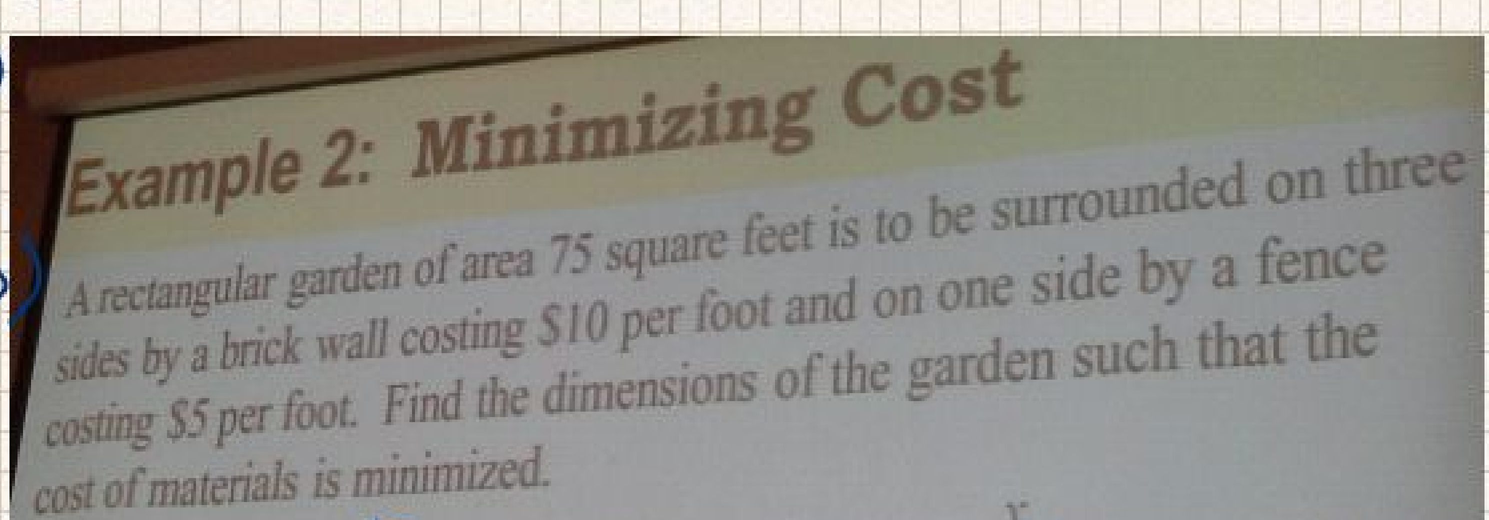
$2l + 2w = 300$

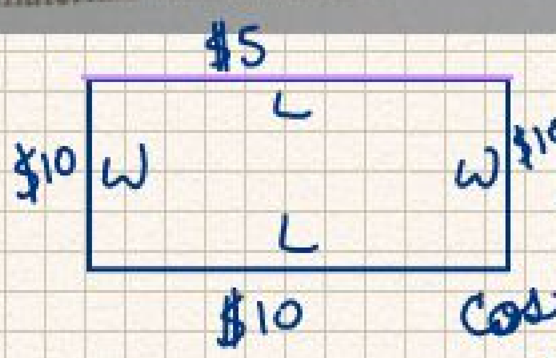
$w = 150 - l$

$A = lw = l(-l + 150)$

$-l^2 + 150l$

$\frac{-b}{2a}$ $\frac{-150}{-2} = 75$





$A = 75\text{ m}^2$ $l = \frac{75}{w}$

$lw = 75\text{ m}^2$

cost $= 10w + 10w + 10l + 5l$

$C = 20w + 15l$

$C = 20w + 15\left(\frac{75}{w}\right)$

$\frac{dc}{dw} = 20 - \frac{15 \cdot 75}{w^2}$

$20 = \frac{1125}{w^2}$ $w^2 = \frac{1125}{20}$

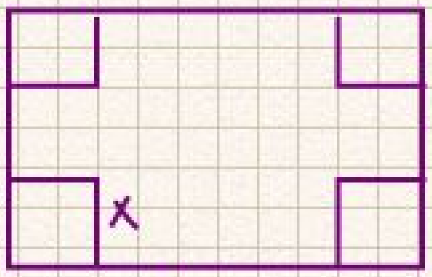
$w = \sqrt{\frac{1125}{20}}$ $w = 7.5\text{ m}$

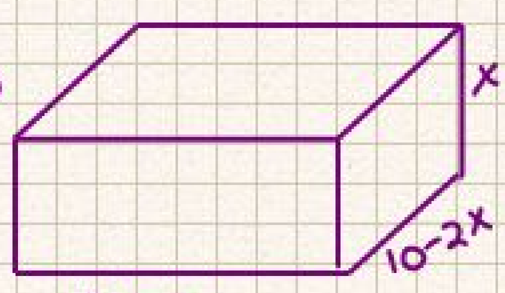
4.6 Worksheet

1-11

+ 4.2/3 finish Ws
4.1 Worksheet
4.1 Syllabus 11/29/2012

Read 4.6 + try 1-11





$l = 16-2x$ ~~8~~ $0 \le X \le 5$ feasible domain

$w = 10-2x$ 5 $V = lwh$

$h = x$ 0 $V = (16-2x)(10-2x)x$

$V = 4x^3 - 52x^2 + 160x$

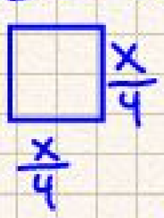
$V' \Rightarrow 12x^2 - 104x + 160$

$-160 < 12x^2 - 104x$

$-40 < 3x^2 - 26x$

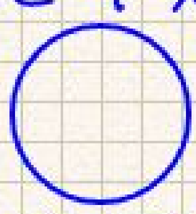
Four Feet of wire is used to form a square and a circle.

Let x = wire used for □



The remaining wire $= 4-x$

$c = 4-x$



$2\pi r = 4-x$

$r = \dfrac{4-x}{2\pi}$

Goal: Maximize A(□+○)

$A = A_\square + A_\circ$

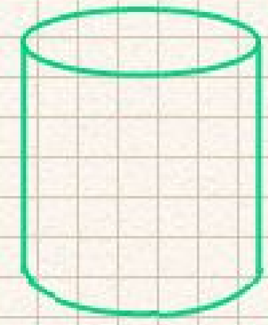
$A = \left(\dfrac{x}{4}\right)^2 + \pi r^2$

$= \dfrac{x^2}{16} + \cancel{\pi}\left(\dfrac{4-x}{2\pi}\right)^2$

$= \dfrac{x^2}{16} + \dfrac{(4-x)^2}{4\pi}$ $0 \le x \le 4$

$\dfrac{dA}{dx}$

HW: Worksheet 4.6
only 1-11, 13-16
12, 17



$V = 1L$

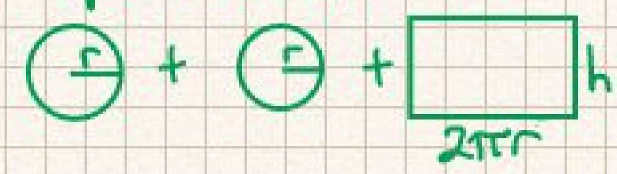
$V = 1000 cm^3$

$= \pi r^2 h = 1000$

$h = \dfrac{1000}{\pi r^2}$

Minimize Surface Area

S = Top + Bottom + Side



$\pi r^2 + \pi r^2 + 2\pi rh$

$2\pi r^2 + 2\pi rh$

$2\pi r^2 + 2\pi r\left(\dfrac{1000}{\pi r^2}\right)$

4.8 Antiderivatives

Find $f(x)$ = given

$f'(x) = x^3 - 4x^2 + 1$

$f(x) = \frac{x^{3+1}}{4} - \frac{4}{3}x^3 + 1x$

anti-derivative

$f'(x) = \frac{1}{x} \Rightarrow f(x) = \ln|x|$

$\lim_{x\to 9} \frac{\sqrt{x^2-3}}{x-9}$

Use l'Hôpitals

$\lim_{\Delta=0} \left(\frac{\sin \Delta}{\Delta}\right) = 1$

$\lim_{x\to\infty} x \cdot \sin\left(\frac{1}{x}\right) =$

$y' = 2x \quad (x,y)$

$y = x^2 + c \quad (1,4)$

General Solution

$4 = 1^2 + c \rightarrow y = x^2 + 3$

Specific Solution

This is called an initial value problem

$1^\infty \quad \frac{0}{0} \quad \infty - \infty \quad 0^0 \quad \frac{\infty}{\infty} \quad \infty \cdot 0 \quad \infty^0$

$\boxed{\text{Homework 4.8} \quad \text{Syllabus 4.6W}}$

3) $\lim_{t\to\infty} \frac{\ln(3t)}{t^2} = \frac{\infty}{\infty}$

$\infty - \infty$ can be $\frac{0}{0}$ or $\frac{\infty}{\infty}$

L'Hospital's $\Rightarrow$

let $f(t) = \ln(3t) \Rightarrow f'(t) = \frac{1}{3t} \cdot 3$

let $g(t) = t^2 \Rightarrow g'(t) = 2t$

$\frac{\frac{3}{3t}}{\frac{2t}{1}} \quad \frac{\frac{A}{B}}{\frac{C}{D}} = \frac{AD}{BC} = \frac{3}{6t^2} = 0$

$\lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{\sin x}\right)$

$= \lim_{x\to 0^+} \left(\frac{\sin x}{x \sin x} - \frac{x}{x \sin x}\right)$

Function	Particular antiderivative	Function	Particular antiderivative
$cf(x)$	$cF(x)$	$\sin x$	$-\cos x$
$f(x) + g(x)$	$F(x) + G(x)$	$\sec^2 x$	$\tan x$
$x^n \ (n \neq -1)$	$\frac{x^{n+1}}{n+1}$	$\sec x \tan x$	$\sec x$
$1/x$	$\ln\lvert x\rvert$	$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x$
e^x	e^x	$\frac{1}{1+x^2}$	$\tan^{-1} x$
$\cos x$	$\sin x$		

$$\lim_{x \to 0} (1+x)^{\frac{1}{x}}$$

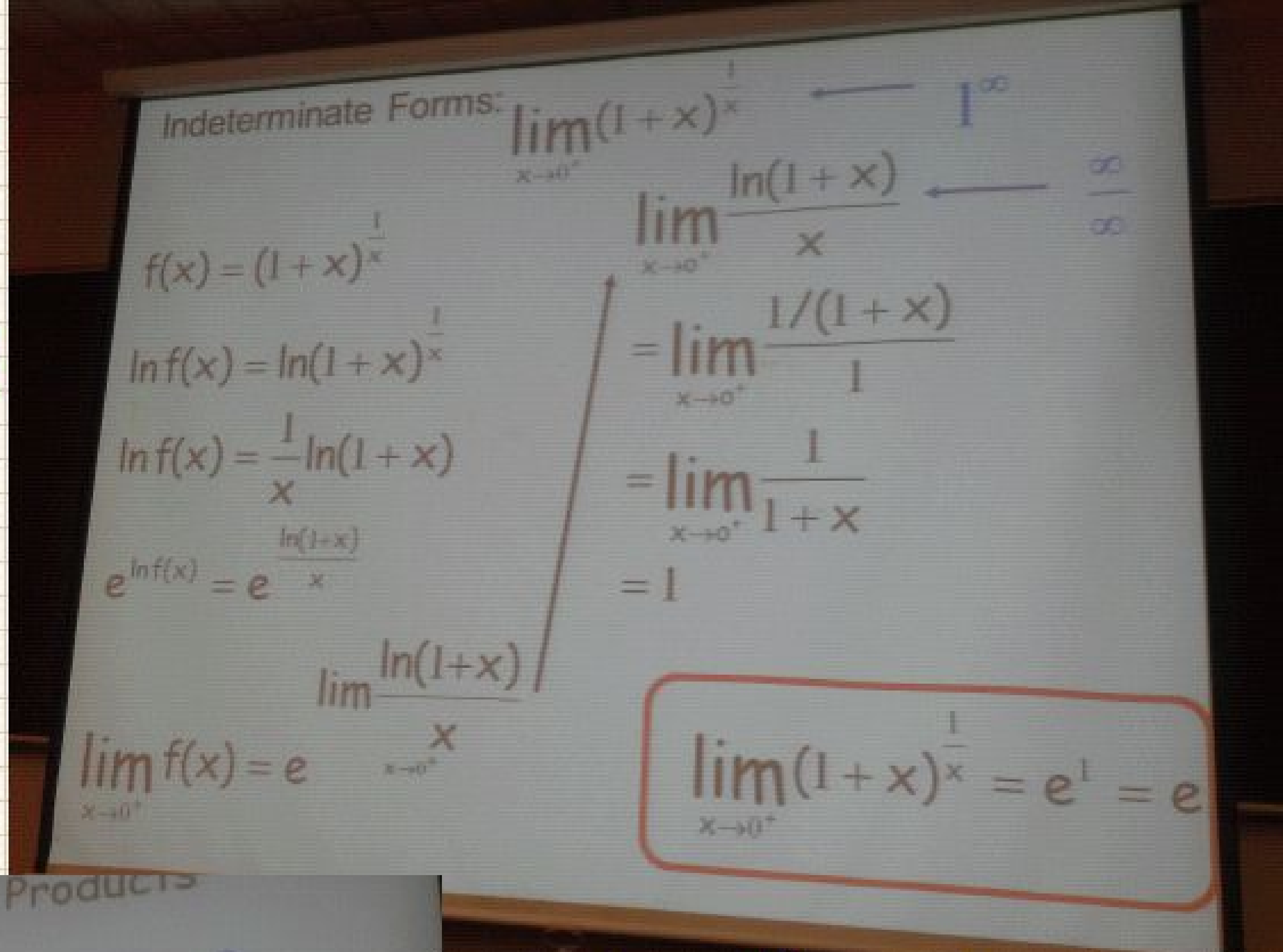
$$\ln f(x) = \ln (1+x)^{1/x}$$

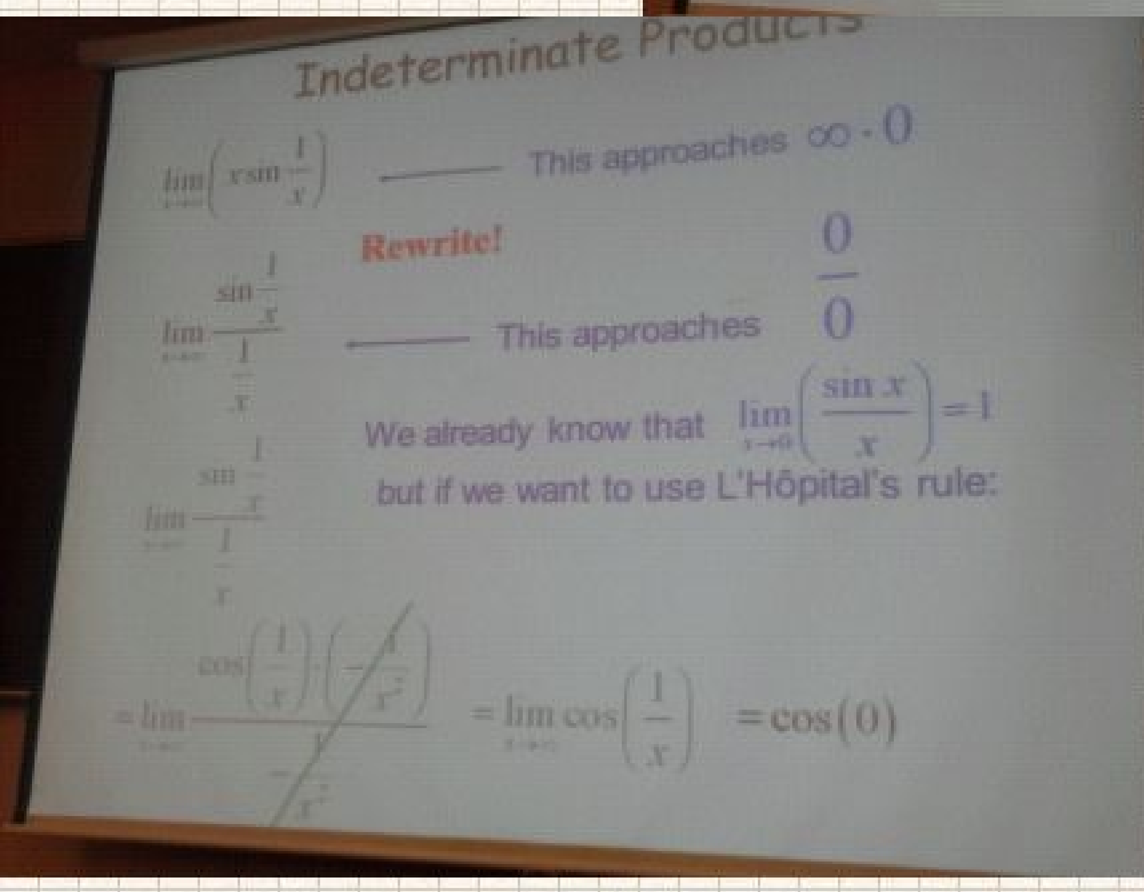
$$\ln f(x) = \frac{1}{x} \ln (1+x)$$

$$\ln f(x) = \frac{\ln(1+x)}{x}$$

$$e^{\ln f(x)} = e^{\frac{\ln(1+x)}{x}}$$

$$\lim_{x \to 0} f(x) = e^{\frac{\ln(1+x)}{x}}$$





$$\lim_{x \to \infty} x^{1/x} \to \infty^0$$

$$f(x) = x^{1/x}$$

$$\ln f(x) = \frac{1}{x} \ln x$$

$$e^{\ln f(x)} = e^{\frac{\ln x}{x}}$$

$$f(x) = e^{\frac{\ln x}{x}}$$